**Modular Representation Theory** **Homework 2**
**Blok 4** **Due date: Mai 23rd 2008**

**1.** Let $G$ be a finite group, let $N$ be a normal subgroup of $G$ and let $k$ be a commutative ring. Show that $J(kN)\,kG = kG\,J(kN)$.

**2.** Let $G$ be a finite group, let $N$ be a normal subgroup of $G$ and let $k$ be a field. Suppose that $\mathrm{char}(k)$ does not divide the index $[G : N]$. Let $M$ be a $kG$-module. Show that $M$ is semi-simple as $kG$-module if and only if $M$ is semi-simple as $kN$-module.

**3.** Let $G$ be a finite group, let $N$ be a normal subgroup of $G$ and let $k$ be a field. Suppose that $\mathrm{char}(k)$ does not divide the index $[G : N]$. Show that $J(kG) = J(kN)\,kG$.

**4.** Let $G$ be a finite group and let $k$ be a field of prime characteristic $p$ such that $p$ divides $|G|$. Show that $J(kG)$ is not zero.

**5.** Let $A$ be a commutative finite-dimensional algebra over a field $k$. Show that $J(A)$ is equal to the set of all nilpotent elements in $A$.

**6.** Give an example of a finite-dimensional algebra $A$ over a field $k$ which has a non-zero nilpotent element not contained in $J(A)$.

**7.** Let $k$ be a field and let $n$ be a positive integer. Let $A$ be the $k$-algebra of all upper triangular $n \times n$-matrices with coefficients in $k$; that is

$$A = \{(a_{ij})_{1\leq i,j\leq n} | a_{ij} \in k, a_{ij} = 0 \text{ if } i > j\}\,.$$

Determine $J(A)$.

**8.** Let $k$ be a field and let $f$ be a polynomial in $k[X]$. When is the quotient algebra $k[X]/fk[X]$ semi-simple?

**9.** Determine the character table of a non abelian group $G$ of order 8.
Hint: There are two isomorphism classes of non abelian groups of order 8, namely the dihedral group $D_8$ and the quaternion group $Q_8$. One doesn't need to know this fact, though, in order to compute the character table; the structural information needed about $G$ which suffices to compute he character table is that $Z(G) = [G, G]$ is of order two and $G/[G, G]$ is a Klein four group.