and on replacing n by $n+l$, a permissible operation,

$$e^{-2l\pi i\beta} H(re^{-2l\pi i\alpha}) = \sum_{\substack{n=-\infty \\ n\neq -l}}^{\infty} \frac{1}{n+l} \frac{e^{2n\pi i\beta}}{1-re^{2n\pi i\alpha}}.$$

Hence

$$e^{-2l\pi i\beta} H(re^{-2l\pi i\alpha}) - H(r) = \sum_{-\infty}^{\infty}{}'' \left(\frac{1}{n+l} - \frac{1}{n}\right) \frac{e^{2n\pi i\beta}}{1-re^{2n\pi i\alpha}} + \frac{1}{l}\frac{1}{1-r} + \frac{1}{l}\frac{e^{-2l\pi i\beta}}{1-re^{-2l\pi i\alpha}},$$

where $n = 0, -l$ are omitted in the summation. Since

$$\sum_n \left(\frac{1}{n+l} - \frac{1}{n}\right)$$

converges absolutely and $H(r) = o\left(1/(1-r)\right)$, we have

$$e^{-2l\pi i\beta} H(re^{-2l\pi i\alpha}) = \frac{1}{l(1-r)} + o\left(\frac{1}{(1-r)}\right);$$

and this is the result (20).

This method does not seem applicable in the case when $xe^{2\pi i\theta} \to 1$ along a radius where θ is a real number such that $\theta = m\alpha + n$ for integers l, m, n.

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AUTOMORPHISM GROUPS OF RESIDUALLY FINITE GROUPS

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1. Suppose that G is a finitely generated residually finite‡ group with automorphism group A. Let $a \in A$, $a \neq 1$, Then for some $g \in G$ $a(g) \neq g$. Put $h = a(g)g^{-1}$. Since $h \neq 1$ there is a normal subgroup of G of finite index not containing h. Now in a finitely generated group there are only finitely many subgroups of fixed finite index n (M. Hall [1]). So there is a characteristic subgroup K of G, of finite index, not containing h. The automorphism group A induces a finite group $\bar{A}$ $(\cong A/L)$ of automorphisms of the finite group G/K. But $h \notin K$; so a induces a non-trivial automorphism of G/K. Consequently $a \notin L$. So A is residually finite. Thus we have proved the following theorem.

THEOREM 1. *The automorphism group of a finitely generated residually finite group is residually finite.*

Theorems of this kind are rare in infinite groups, since properties of a group are seldom inherited by its automorphism group. It is, therefore, not surprising that Theorem 1 has many interesting applications. It suffices, perhaps, to give three of these.

2. Let R be any ring with a unit element which is additively a finitely generated abelian group. Then any group of matrices over R is residually finite, by Theorem 1, since it can be viewed as a group of automorphisms of a finitely generated abelian group. In particular, then, the integral unimodular group is residually finite. But it is easy to see that a free group of infinite rank is one of the subgroups of the unimodular group. So our argument has yielded a simple new proof of a theorem of Levi [2] which asserts that a free group is residually finite.

3. If F is a finitely generated free group, then its automorphism group ϕ is residually finite. But ϕ is finitely generated (Nielsen [3]). So its automorphism group Φ is residually finite. It is perhaps worth pointing out it is not easy to prove either ϕ or Φ residually finite by a direct procedure, starting out from given presentations of them.

4. Polycyclic and finitely generated metanilpotent groups are residually finite (Hirsch [4] and P. Hall [5], respectively). So their automorphism groups are residually finite.

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‡ G is residually finite if there corresponds to each $x \in G$ $(x \neq 1)$ a normal subgroup N_x of G such that $x \notin N_x$ and G/N_x is finite.

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References.

1. M. Hall, " Subgroups of finite index in free groups ", *Canadian J. of Math.*, 1 (1949), 187–190.
2. F. W. Levi, " Über die Untergruppen der freien Gruppen (II) ", *Math. Zeitschrift*, 37 (1933), 90–97.
3. J. Nielsen, " Die Isomorphismengruppe der freien Gruppen ", *Math. Annalen*, 91 (1924), 169–209.
4. K. A. Hirsch, " On infinite soluble groups (III) ", *Proc. London Math. Soc.*, 49 (1946), 184–194.
5. P. Hall, " The Frattini subgroups of finitely generated groups ", *Proc. London Math. Soc.* (3), 11 (1961), 327–352.

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NOTE ON A PROBLEM IN ELASTODYNAMICS

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1. *Introduction*

In this note the techniques developed by the author [**1**, **2**] for reducing certain boundary value problems to the solution of a Fredholm integral equation of the second kind are applied to a problem in elastodynamics. The problem considered is that of the torsional oscillations set up in an elastic stratum, one face of which is rigidly secured, by a rigid circular disk which is attached to the free surface of the material and which performs simple harmonic oscillations about its axis. This problem has recently been considered by Collins [**3**] who also reduces it to a Fredholm integral equation of the second kind and obtains an approximate solution by iteration when the frequency is small and the depth of the stratum large. The integral equation obtained here is not identical with that derived by Collins, the discrepancy is due to the existence of a slight error in the analysis of [**3**].

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2. *Solution of boundary value problem.*

Cylindrical polar coordinates (ρ, ϕ, z) are chosen so that the elastic stratum occupies the region $0 \leqslant z \leqslant f$. A disk of unit radius is assumed attached to the portion $0 \leqslant \rho \leqslant 1$ of the free surface $z = f$, the disk performing simple harmonic oscillations of amplitude Φ and period $2\pi/\omega$ about its axis. The density and shear modulus of the elastic material will be denoted by ρ_0 and μ respectively.

The boundary value problem reduces [**3**] to the solution of

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial v}{\partial v} - \frac{v}{\rho^2} + \frac{\partial^2 v}{\partial z^2} + k^2 v = 0, \tag{1}$$

where $k^2 = \rho_0 \omega^2/\mu$, subject to the conditions,

$$v(\rho, 0) = 0, \quad v(\rho, f) = \Phi\rho, \quad 0 \leqslant \rho \leqslant 1,$$

$$\frac{\partial v}{\partial z}(\rho, z) = 0 \text{ on } z = f, \quad \rho \geqslant 1.$$

v is required to be finite and continuous at all points within the stratum and $\partial v/\partial z$ is assumed to have a singularity of order $(1-\rho^2)^{-\frac{1}{2}}$ near the edge of the disc. At a large distance from the z axis v is required to represent a disturbance travelling outwards.

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