Algorithmic consequences of the Linearly Bounded Conjugator Property in braid groups

"Garside theory; state of the art and prospects" - Cap Hornu

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Both CDP and CSP are solvable in braid groups (Garside, 1969).

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- In this talk, n will be fixed and l will be the only parameter.

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Theorem (C., Wiest)

There is an algorithm for solving the CDP and CSP in the 4-strand braid group B_4 whose complexity depends cubically on the length of the input.

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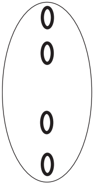

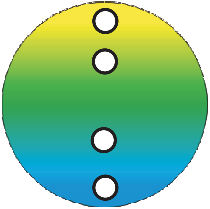

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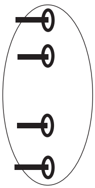

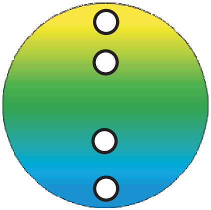

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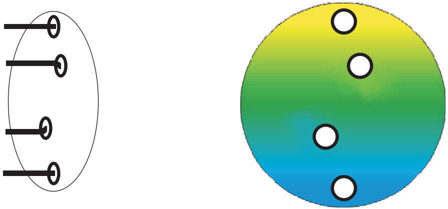

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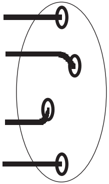

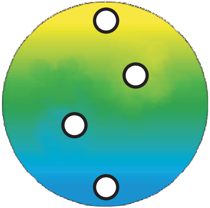

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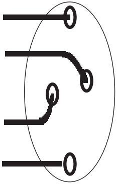

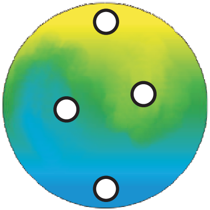

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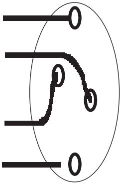

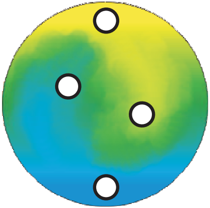

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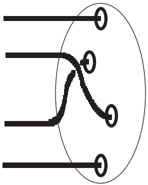

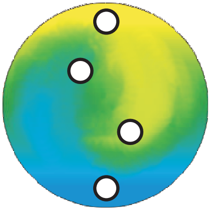

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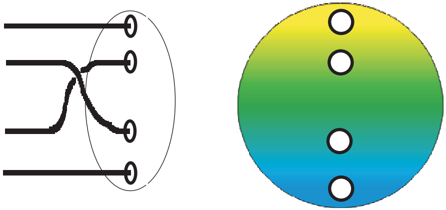

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Theorem (Thurston, 1980's)

Any braid $x \in B_n$ is either

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The Nielsen-Thurston type is invariant under conjugation and taking powers.

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- In the reducible case, one can try to solve CDP and CSP by gluing "irreducible" pieces together.

The Linearly Bounded Conjugator Property

Theorem (Masur-Minsky, 2000)

Let n be a positive integer. Choose a generating set $\mathcal{G}_n$ for B_n. There exists a constant $C(\mathcal{G}_n)$ such that for any pair $x, y \in B_n$ of pseudo-Anosov conjugate braids, one can find a conjugator u between them satisfying

$$|u|_{\mathcal{G}_n} \leqslant C(\mathcal{G}_n) \cdot (|x|_{\mathcal{G}_n} + |y|_{\mathcal{G}_n}).$$

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The constant C is NOT explicitly known.

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Define a special kind of conjugation, called “cyclic sliding” and denoted $\mathfrak{s}$ (Gebhardt & González-Meneses, 2008).

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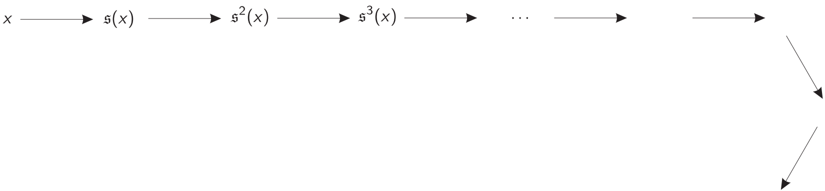

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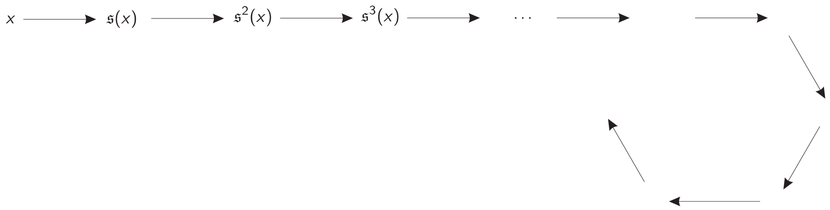

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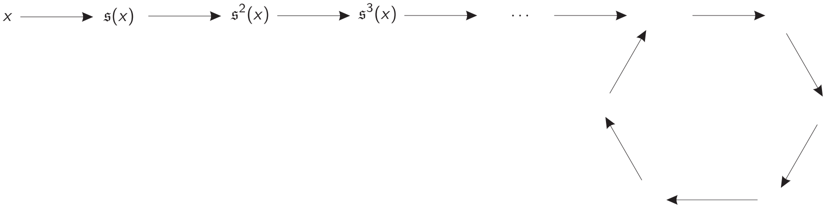

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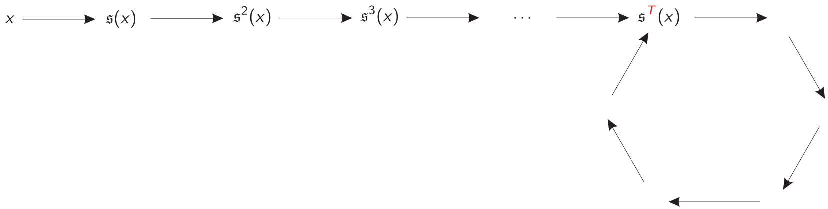

The set of sliding circuits

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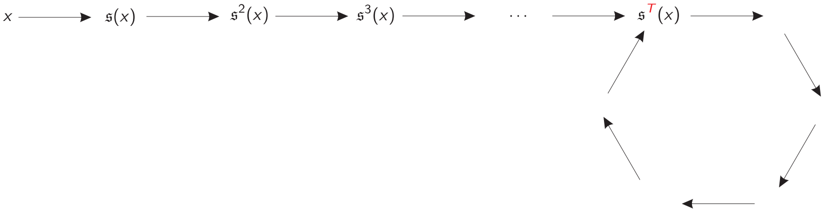

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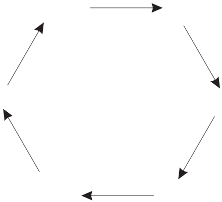

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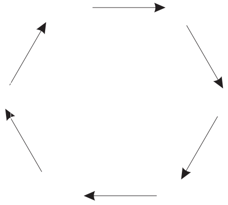

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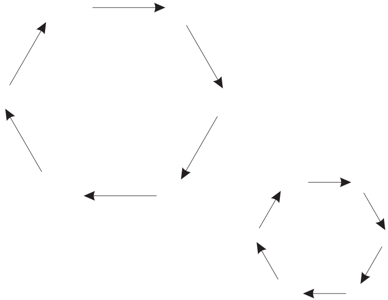

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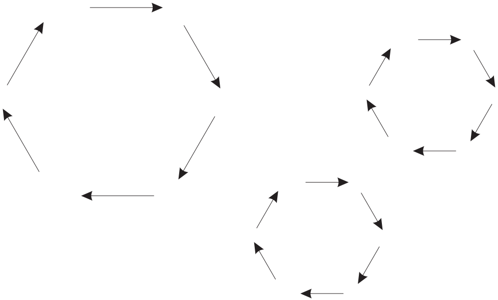

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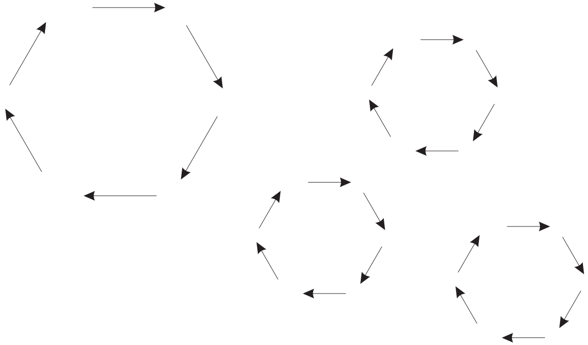

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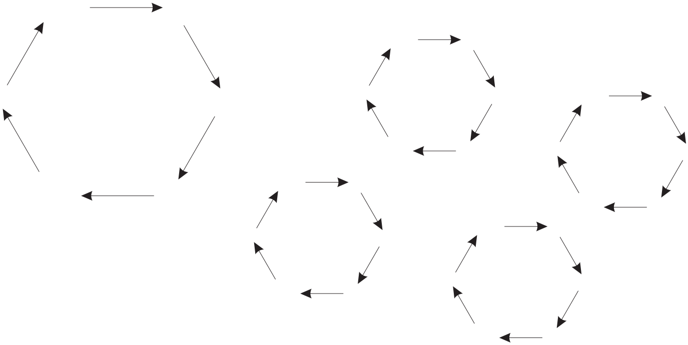

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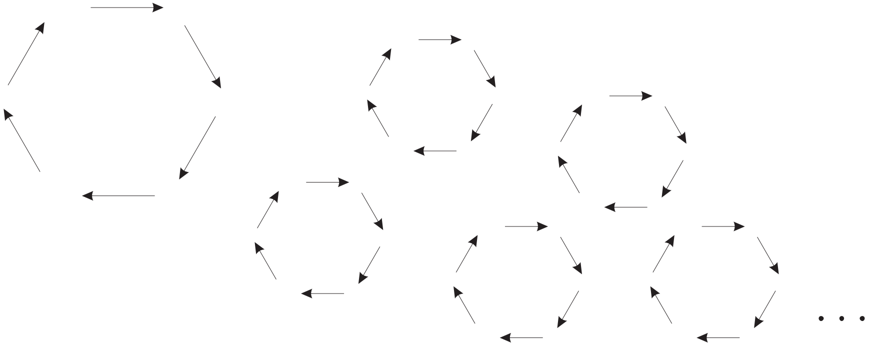

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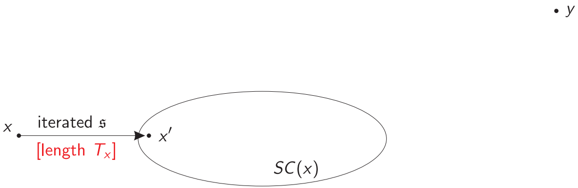

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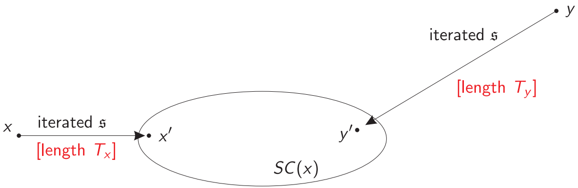

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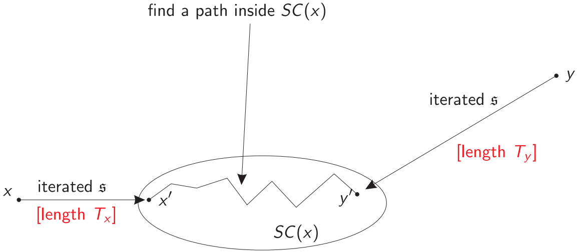

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We want to answer in the pseudo-Anosov case.

The special case of rigid elements

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Moreover, rigidity is easy to check (just compute the normal form).

In general, the SC's of rigid elements have rather simple structure, although some difficulties may appear...

A (partial) solution to CDP/CSP for pA braids

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Theorem

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In general, no polynomial bound (in l and n) on $\#SC$, for pA rigid braids (Prasolov).

Proof

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Lemma

Given two pseudo-Anosov braids x and y, we can produce effectively $\bar{x}, \bar{y}$ pA rigid s.t.

- $x \sim y \Longleftrightarrow \bar{x} \sim \bar{y}$,
- *if so, the knowledge of a conjugator* $\bar{x} \longrightarrow \bar{y}$ *implies the knowledge of a conjugator* $x \longrightarrow y$,
- $length(\bar{x}) = O(length(x))$, $length(\bar{y}) = O(length(y))$.

The use of the linear bound

Theorem

Let x be a pseudo-Anosov braid. Suppose that x has some rigid conjugate. Then T_x is bounded above by $C \cdot \mathrm{length}(x)$. In particular, $\mathfrak{s}^{C \cdot \mathrm{length}(x)}(x)$ is rigid.

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This gives a non-explicit linear bound on T above in the pA rigid case.

Passing to powers

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Theorem (Birman, Gebhardt, G.-Meneses)

For fixed n, there exists a (explicit) polynomial $K(n)$ s.t. for any pA n-braid x, there exists a power $m_x \leqslant K(n)$ with x^{m_x} conjugate to a rigid.

Main step

Theorem

There exists an algorithm of complexity $O(l^2)$ with:

- *INPUT: $x, y \in B_n$ pA (of length at most l),*
- *OUTPUT: $s \in \mathbb{N}$, $\bar{x}, \bar{y}, \widetilde{x}, \widetilde{y} \in B_n$ s.t.*

$$x^s \xrightarrow{\widetilde{x}} \bar{x} \qquad\qquad y^s \xrightarrow{\widetilde{y}} \bar{y}$$

with $\bar{x}$, $\bar{y}$ rigid, s bounded independently of length(x), length(y).

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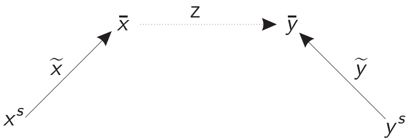
Finally, the previous algorithm also gives $\tilde{x}$ and $\tilde{y}$ s. t.



Description of the latter algorithm

Description of the latter algorithm

x

Description of the latter algorithm

x

x^2

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x

x^2

x^3

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x

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$\vdots$

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$x^{K(n)}$

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Description of the latter algorithm

$$x \longrightarrow \mathfrak{s}(x) \longrightarrow \mathfrak{s}^2(x)$$

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- $\tilde{x}$, $\tilde{y}$ are obtained as the product of arrows involved in cyclic slidings.

Solution to CDP/CSP in B_4

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Corollary

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- the edges still correspond to "minimal conjugators".

Structure of SC's of rigid elements

Remark: We use the Birman-Ko-Lee structure.

The set of Sliding circuits is endowed with the structure of a connected directed graph.

The vertices are the elements of SC, the edges some "minimal conjugators".

In the rigid case, results by Gebhardt about the "transport" allow to consider this graph modulo conjugation by δ and cycling.

We obtain the quotient graph $SC_{\sim}(x)$:

- each vertex corresponds to an equivalence class containing $4 \cdot length(x)$ elements of $SC(x)$
- the edges still correspond to "minimal conjugators".

We need to bound linearly (w.r.t. the length) the number of vertices of $SC_{\sim}(x)$.

The quotient graph

Thanks to the simplicity of the lattice of simple elements in the Birman-Ko-Lee structure of B_4, one can show that this quotient graph $SC_{\sim}(x)$ has one of the following forms.

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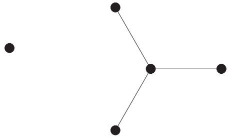

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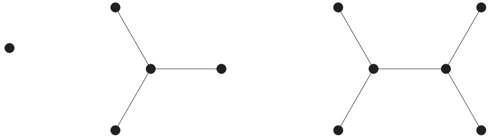

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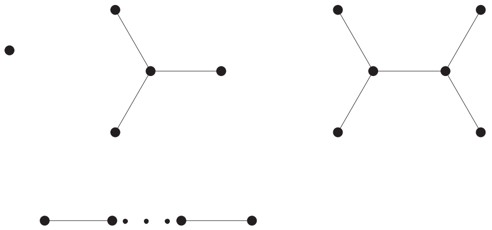

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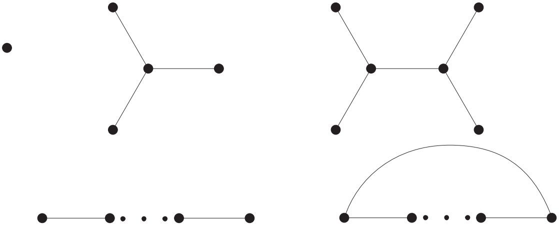

Bounding the line

As edges are given by minimal conjugators we can use again Masur-Minsky's bound: the length of the line is linearly bounded by the length of the braid we started with.

Another (not explicit) polynomial-time algorithm

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Theorem (C.)

There exists an algorithm which decides the Nielsen-Thurston type of a given braid on n strands and of length l in time $O(l^3)$ *for each fixed n.*

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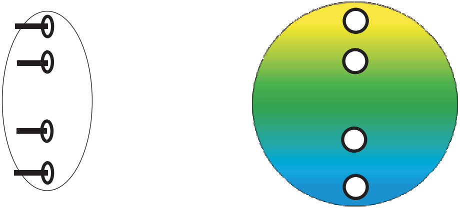
5) Otherwise, for the rigid element $\widetilde{x}$ obtained, one can test in an effective way whether it is reducible or pseudo-Anosov (G.-Meneses, Wiest).

Questions

- Look at the geometry of the curve complex associated to the n-times punctured disk and find the value of C.
- Does LBC hold in Garside groups?



[1]

Thank you

[1] This picture by courtesy of Marta Aguilera.