Matrices in polynomial system solving

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This talk is about the role of matrix computations in the problem of solving systems of polynomial equations. Let k be an algebraically closed field and let $p_1 = p_2 = \ldots = p_n = 0$ define such a system in k^n: $p_i \in k[x_1, \ldots, x_n]$. Let I be the ideal generated by these polynomials. We are interested in the case where the system has finitely many isolated solutions in k^n. It is a well known fact that this happens if and only if the quotient ring $k[x_1, \ldots, x_n]/I$ is finite dimensional as a k-vector space. The multiplication endomorphisms of the quotient algebra provide a natural linear algebra formulation of the root finding problem. Namely, the eigenstructure of the multiplication matrices reveals the solutions of the system. These multiplication matrices can be calculated from the coefficients of the p_i, for example by using Groebner bases. The computations make an implicit choice of basis for $k[x_1, \ldots, x_n]/I$, which from a numerical point of view is not a very good choice. Significant improvement can be made by using elementary numerical linear algebra techniques on a Macaulay-type matrix. In this talk we will present this technique and show how the resulting method can handle challenging systems.