What is Non-Linear Preconditioning ?

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The idea of preconditioning iterative methods for the solution of linear systems goes back to Jacobi (1845), who used rotations to obtain a system with more diagonal dominance, before he applied what is now called Jacobi's method. The preconditioning of linear systems for their solution by Krylov methods has become a major field of research over the past decades, and there are two main approaches for constructing preconditioners: either one has very good intuition and can propose directly a preconditioner which leads to a favorable spectrum of the preconditioned system, or one uses the splitting matrix of an effective stationary iterative method like multigrid or domain decomposition as the preconditioner.

Much less is known about the preconditioning of non-linear systems of equations. The standard iterative solver in that case is Newton's method (1671) or a variant thereof, but what would it mean to precondition the non-linear problem ? An important contribution in this field is ASPIN (Additive Schwarz Preconditioned Inexact Newton) by Cai and Keyes (2002), where the authors use their intuition about domain decomposition methods to propose a transformation of the non-linear equations before solving them by an inexact Newton method. Using the relation between stationary iterative methods and preconditioning for linear systems, we show in this presentation how one can systematically obtain a non-linear preconditioner from classical fixed point iterations, and present as an example a new two level non-linear preconditioner called RASPEN (Restricted Additive Schwarz Preconditioned Exact Newton) with substantially improved convergence properties compared to ASPIN.