Core-Chasing Algorithms for the Eigenvalue Problem

David S. Watkins
Department of Mathematics
Washington State University
watkins@math.wsu.edu

If A is an $n \times n$ upper Hessenberg matrix, the decomposition $A=QR$ is particularly simple: the unitary matrix Q is a product of a descending sequence of $n-1$ core transformations (e.g.\ Givens rotations) $Q_{1} \cdots Q_{n-1}$. Thus the space required to store the decomposed form is not appreciably greater than that required to store A in the conventional way. Forcertain classes of structured eigenvalue problems we (and others before us) have found it useful to store A in this QR-decomposed form and operate on the factors. We mention in particular the unitary-plus-rank-one case, which includes companion matrices, and the even simpler unitary case.

In this talk we will argue that even if A has no special structure, it is still worthwhile to do the QR decomposition and work with the factored form. If we apply Francis's implicitly-shifted QR algorithm to the matrix in this form, it becomes a process of chasing an unwanted core transformation through the matrix until it is eliminated at the bottom. Thus it is a core-chasing algorithm instead of a bulge-chasing algorithm.

This is joint work with Jared Aurentz, Thomas Mach, Leonardo Robol, and Raf Vandebril.