

MASOH PROJECT
MODÉLISATION ANALYSE SIMULATION
D'ÉQUATIONS D'ONDES HYDRODYNAMIQUES

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1. Original idea and aims
2. Organization, managing of the network (missions, meetings, diffusion)
3. Balance
4. Results
5. Conclusion
6. Next ?

SYNERGY OF EXISTING COLLABORATIONS ON A TOPIC

Understanding phenomena of water wave propagation

BACKGROUND

- ▶ Axe Amiens Marrakech (Nonlinear dispersive damped wave equations)
Asymptotic behavior, asympt. regularization; Scientific stays; Articles, PhD co-advising; CNRS (France) / CNRST (Maroc) grants
- ▶ since 2000 Axe Lille-Orsay-Monastir
Multilevel numerical schemes for Fluid dynamics; development of numerical schemes for dispersive eqs (KdV) ; Scientific stays; Articles ; CNRS/DGRST grant (since 2003) + CMCU ; European educational grant TEMPUS
- ▶ Axe Lille-Granada
Kinetic models (Schrödinger-Poisson like); Scientific stays ; Collaborations Th. Goudon - Granada team

WATER WAVE PROPAGATION

OBJECTIVES

- ▶ Numerical modeling of dispersive wave eqs (KdV ...)
- ▶ Numerical schemes for dispersive phenomena
- ▶ Multilevel schemes
- ▶ Mathematical and numerical analysis of the schemes
- ▶ Validation
- ▶ Tool for interaction with modeling and mathematical analysis of these equations

QUANTITATIVE RESULTS

- ▶ Articles : 4 accepted /published , 5 submitted / in preparation
- ▶ 3 MASOH meetings (with external speakers)
- ▶ Mini-symposium session at CANUM 2008
- ▶ 4 talks given in international conferences
- ▶ 5 co-advised Phd thesis
- ▶ construction of a scientific network by using missions
- ▶ use of other resources (academic invitations, compatibles programs (TEMPUS, Tunisia))

SCIENTIFIC BALANCE

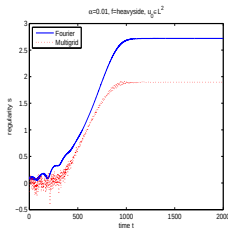
- ▶ Some Scientific results
- ▶ New enlarged Network
- ▶ publications
- ▶ Advising of PhD Students
- ▶ organization of meetings
- ▶ Diffusion on meeting / Internet

WEAKLY DAMPED SCHRÖDINGER EQUATION

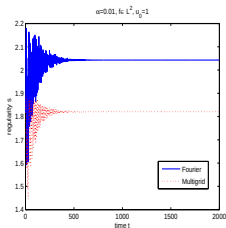
$$u_t + \alpha u + i(u_{xx} + |u|^2 u) = f, \quad u(x, t) \in \mathbb{R}_t \times \mathbb{T}_x$$

$$u \in H^s \iff \sum_{k=1} 4^{ks} \|z_k\|_{L^2}^2 < +\infty \quad (\text{Hierarchical basis})$$

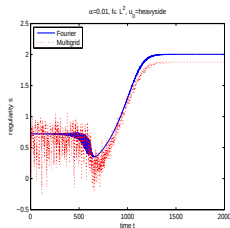
$$u \in H^s \iff \sum_{N=1}^{+\infty} N^{2s-1} \|u_N - u_{N/2}\|_{L^2}^2 < +\infty \quad (\text{Fourier})$$



(a)



(b)



(c)

FIGURE: Weakly damped Schrödinger, numerical measure of the asymptotic time

THESIS I. DAMERGI

Luc Bergé (CEA) proposed to stabilize blowing up soliton by adding an oscillating term Numerical of 2D oscillating NLS equation

$$i\partial_t u + \Delta u + 2\cos^2(\Omega t)|u|^2 u = 0$$

Tracking of blow up of the solution for negative energy initial data ; role of Ω stabilizing term (blow up time increases with Ω for $\Omega < \Omega_c$, for $\Omega \geq \Omega_c$ no blow up observed)

Article (I.D + OG) to be published in J. Math. Anal. : study of possible blow up in finite time in $H^1(\mathbb{R}^2)$ for solutions to critical and supercritical nonlinear Schrödinger equations with an oscillating nonlinearity.

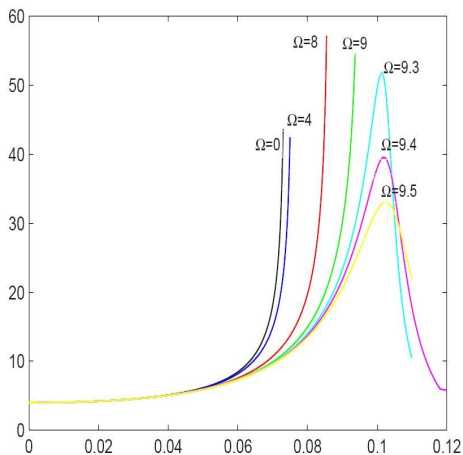


FIGURE: Time evolution of the blow up time for different values of Ω , $N=500$
(a)

THESIS E. EZZOUG

SEMI-DISCRETE WEAKLY DAMPED NONLINEAR
2-D SCHRODINGER EQUATION

$$u_t + \alpha u + i\Delta u + i|u|^2 u = f(x) \quad x \in \mathbb{T}^2 \quad u(0; x) = u_0(x); x \in \mathbb{T}^2$$

Numerical simulations + proof of the existence of a finite dimensional global attractor for this dynamical system (proven in 1D, in 2D : new !)

SOME WORK IN PROGRESS

ACTUALLY

- ▶ Weakly Damped KdV equation (MASOH, M.A. H.M, C.C, J.P. C.)
 - ▶ Simulation of nontrivial dynamics (Hopf bifurcations)
 - ▶ Splitting of the solution on regular / non regular part and multilevel scheme for solving regular component
- ▶ Thesis of M. Moalla (Numerical splitting scheme for quasi-geostrophic equations)
- ▶ Multilevel stabilization of wave-like equation (CNRS/LEA, JPC +L. Ignat (Inst. Stoilow Bucarest))

A NEW AND ENLARGED SCIENTIFIC NETWORK

- ▶ Lille-Amiens-Orsay Monastir
(O. Goubet, C. Calgaro, C. Besse, J.-P. Chehab, L. Di Menza, J. Laminie) (E. Zahrouni, I. Damergi, E. Azzoug) M. Abounouh,
- ▶ Lille-Amiens Marrakech (O. Goubet, C. Calgaro, C. Besse, J.-P. Chehab, S. Dumont) (M. Abounouh, H. Al Moatassime)
- ▶ Marrakech Monastir
E. Zahrouni M. Abounouh
- ▶ Lille - Amiens Granada (Th. Goudon) (J. Nieto, J Soler) + contact
(L. Dupaigne, J. Soler)

Remark: Lille-Amiens (Orsay) = Cluster

ARTICLES (ACCEPTED / REVISION)

1. A. Abounouh, H. Al Moatassime, J.-P. Chehab, S. Dumont, O. Goubet, Discrete Schrödinger Equations and dissipative dynamical systems, Comm. in Pure and Applied Analysis, 7, (2008), no 2, 211-227.
2. I. Damergi et O. Goubet, Blow up solutions to the nonlinear Schrodinger equation with oscillating nonlinearities, Journal of Math. Analysis and Applications, in Press
3. O. Goubet et E. Zahrouni, On a time discretization of a weakly damped forced nonlinear Schrodinger equation, Communication in Pure and Applied Analysis, 7 (2008) 14291442
4. C. Calgaro, J.-P. Chehab, J. Laminie, E. Zahrouni, Schémas multiniveaux pour les équations d'ondes, in revision at PROC ESAIM

ARTICLES (SUBMITTED / IN PREPARATION)

1. M. Abounouh, H. Al Moatassime, C. Calgaro, J.-P Chehab, Global attractor for a total discretization of damped forced Korteweg-de Vries equation, in preparation
2. E. Ezzoug , O. Goubet, E. Zahrouni, Semi-Discrete Weakly Damped Nonlinear 2-D Schrodinger Equation. submitted.
3. E. Zahrouni R. May, Global Existence Solution for Subcritical Quasi-Geostrophic Equations. submitted to Nonlinear Analysis
4. M. Abounouh et E. Zahrouni, Semi-Discrete Weakly Damped Nonlinear for Subcritical 2-D Schrodinger Equation, in preparation
5. J. J. Nieto, O. Sanchez, J. Soler, High field regimes and non standard shock relations in semiconductor superlattices theory, in preparation

+ many others to be written !!

PHD CO-ADVISING

1. Since July 2006, co-advising of Emna Ezzoug O. Goubet with E. Zahrouni, J. Laminie
2. Since July 2006, co-advising of Ibtissem Damergi with E. Zahrouni and Ch. Besse
3. Since July 2008 co-advising of Maithem Moalla (Trabelsi) and E. Zahrouni et J.P Chehab
4. Wided Kechiche (O. Goubet, E. Zahrouni) first inscription en 2006-2007) : Etude de systèmes d'équations de Schrödinger non-linéaires.
5. Brahim Alouini (O. goubet E.Z.) Equation de Bose-Einstein sur un canal.

NEW THESIS IN A MASOH'S TOPIC

- ▶ G. Sadaka (LAMFA), oct 2008 (dir. J.-P. C, Picardie Regional Grant)
Etude mathématique et numérique d'équations d'ondes aquatiques

SCIENTIFIC MEETING AND DIFFUSION

MASOH DAYS

1. December 2006

- ▶ Jean-Claude SAUT (Orsay)
Systèmes de Boussinesq faiblement transverses,
- ▶ Laurent di MENZA (Paris XI)
Calcul numérique de solitons

2. June 11 2007 , second MASOH day in Lille.

- ▶ Yvan Martel (Université de Versailles St Quentin)
Collision de solitons pour $gKdV$ non intégrable
- ▶ Luc Bergé (CEA DAM, Bruyères le Chatel)
Modèles de propagation pour les impulsions optiques ultra courtes

3. december, 14 2007 Third MASOH day in Lille.

- ▶ Anne De Bouard (CMAP, Ecole Polytechnique, Palaiseau)
Propagation des solitons pour l'équation de Korteweg-de Vries en présence de fluctuations aléatoires
- ▶ Frédéric DIAS (CMLA, ENS Cachan)
Sur la modélisation des vagues extrêmes et des Tsunamis

MINI-SYMPOSIUM MASOH AT CANUM (MAY 2008)

- ▶ Mostafa Abounouh (FST Marrakech, Maroc) *Attracteur global pour l'équation de Korteweg-de Vries amortie semi-discrétisée en temps*
- ▶ Laurent di Menza (Orsay and Reims since 1/09/08)
Etats localisés : de l'optique non linéaire aux modèles gravitationnels
- ▶ Youcef Mammeri(USTL)
Etude numérique du comportement en temps long des solutions d'équations de Kadomtsev-Petviashvili généralisées en dimension 3
- ▶ Dennis Dutykh (CMLA, ENS Cachan, since 1/09/08 CNRS Chambéry) *Simulation numérique dans l'hydrodynamique côtière*

TALKS GIVEN IN AN INTERNATIONAL CONGRESS

- ▶ "Global Existence Solution for Subcritical Quasi-Geostrophic Equations", conference "fractals and related fields", September 2007 Monastir, Tunisie. E. Zahrouni
- ▶ "Discrete nonlinear Schrodinger equations. Dynamical Systems and numerics", AIMS conference, May 2008, Arlington, USA. OG
- ▶ "Finite Hausdorff dimension for the global attractor of some nonlinear Schrödinger flow", conference "fractals and related fields", September 2007 Monastir, Tunisia. OG
- ▶ "Damped forced nonlinear Schrodinger equations", Imacs conference nonlinear phenomena and wave equations, Athens, USA, April 2007. OG

DIFFUSION

WEB SITE

<http://www.lamfa.u-picardie.fr/chehab/MASOH/masoh.html>

CONCLUSIONS

- ▶ The financial and administrative support provided by INRIA is a very good tool for building a network. First step before considering huger partnerships
- ▶ Research / scientific formation
- ▶ Other partners, non necessarily in mediterranean countries, can be connected to this network on the same topic (CNRS Partnerships with Bucarest)
- ▶ other programs can (must) be used form supporting the network in Studend formation such as Peaople IRSES,
- ▶ little limitation: teaching duties. Mainly, the actors of MASOH are teachers **and** researchers. Delegations at INRIA can help ...

THE FUTURE

- ▶ Ideally a prolongation of the project on a close topic for
 - ▶ achieving (making in progress) the original scientific program
 - ▶ including new partners
- ▶ Prepare applications for larger programs