Functorial equivalence of blocks

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Representations of Finite Groups
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Categories

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$$\forall (h,g) \in H \times G,\ \forall m \in M,\ h \cdot m \cdot g := (h, g^{-1}) \cdot m.$$

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  then $N \otimes_{kH} M$ is a diagonal p-permutation (kK, kG)-bimodule.

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This is equivalent to saying that there exists $\sigma \in cRT^\Delta(H, G)b$ and $\tau \in bRT^\Delta(G, H)c$ such that $\sigma \circ \tau = [kHc]$ and $\tau \circ \sigma = [kGb]$.

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4. *The functor $\mathbb{F}T^{\Delta}_{G,b}$ is isomorphic to $\mathbb{F}T^{\Delta}_{D}$.*

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$$\mathrm{k}(kGb) - \mathrm{l}(kGb) = \mathrm{k}(kHc) - \mathrm{l}(kHc).$$

THANK YOU!

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The category $\mathcal{C}$ is equivalent to the category of finite dimensional K-vector spaces.

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$$\bigoplus_{(P,e_P)\in[\mathcal{F}_b]} \bigoplus_{\pi\in[\mathcal{P}_{(P,e_P)}(L,u)]} \mathbb{F}\mathrm{Proj}(ke_P C_G(P), u) \otimes_{\overline{Aut(L,u)_{(P,e_P,\pi)}}} V,$$

where $\mathrm{Proj}(ke_P C_G(P), u) \leq \mathrm{Proj}(ke_P C_G(P))$ is generated by u-invariant indecomposable modules

Multiplicity formula 2

Let $\mathcal{F}_b$ denote the fusion system of kGb with respect to a maximal b-Brauer pair (D, e_D). For each subgroup $P \leq D$, let e_P denote the unique block of $kC_G(P)$ with $(P, e_P) \leq (D, e_D)$.
For $(P, e_P) \in \mathcal{F}_b$, let $\mathcal{P}_{(P,e_P)}(L, u)$ denote the set of group isomorphisms $\pi : L \to P$ with $\pi i_u \pi^{-1} \in Aut_{\mathcal{F}_b}(P, e_P)$. The set $\mathcal{P}_{(P,e_P)}(L, u)$ is an $(N_G(P, e_P), Aut(L, u))$-biset via $g \cdot \pi \cdot \varphi = i_g \pi \varphi$, for $g \in N_G(P, e_P)$, $\pi \in \mathcal{P}_{(P,e_P)}(L, u)$ and $\varphi \in Aut(L, u)$. Let $[\mathcal{P}_{(P,e_P)}(L, u)]$ denote a set of representatives of orbits.

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where $\mathrm{Proj}(ke_P C_G(P), u) \leq \mathrm{Proj}(ke_P C_G(P))$ *is generated by u-invariant indecomposable modules, and*

$$Aut(L, u)_{\overline{(P,e_P,\pi)}} = \{\varphi \in Aut(L, u) \mid \exists g \in N_G(P, e_P),\ \pi\varphi\pi^{-1} = i_g\}.$$

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Theorem

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$$\bigoplus_{(P_\gamma,\pi)\in[\mathcal{L}_b(G,L,u)]} V^{Aut(L,u)_{\overline{(P_\gamma,\pi)}}}.$$

←