Diagonal p-permutation functors and blockwise Alperin's weight conjecture

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International Workshop on Algebra and Representation Theory
in honor of Alexander Zimmermann

ECNU, Shanghai

Joint work with Deniz Yılmaz

Joint work with Robert Boltje and Deniz Yılmaz

Categories

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Category: Objects + Morphisms.

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Finite groups

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Aim: Build a category of finite groups with other types of morphisms? Using group algebras and bimodules?

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For a block idempotent b of kG, a minimal subgroup D of G such that there exists $c \in (kG)^D$ with $b = \sum_{g \in G/D} gcg^{-1}$ is called a defect group of b. We denote it by $D(b)$.

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$$\forall (h, g) \in H \times G,\ \forall m \in M,\ h \cdot m \cdot g := (h, g^{-1}) \cdot m.$$

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  Examples:

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*A diagonal p-permutation functor over R is an R-linear functor from Rpp_k^{Δ} to R-**Mod**. These functors form an abelian category $\mathcal{F}_{Rpp_k}^{\Delta}$.*

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This is equivalent to saying that there exists $\sigma \in cRT^{\Delta}(H, G)b$ and $\tau \in bRT^{\Delta}(G, H)c$ such that $\sigma \circ \tau = [kHc]$ and $\tau \circ \sigma = [kGb]$.

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Alperin's weight conjecture (with R. Boltje and D. Yılmaz)

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The set $\mathcal{L}_b(G, L, u)$ is a $(G, Aut(L, u))$-biset via $g \cdot (P_\gamma, \pi) \cdot \varphi = ({}^g P_{{}^g\gamma}, i_g \pi \varphi)$, for $g \in G$ and $\varphi \in Aut(L, u)$. Let $[\mathcal{L}_b(G, L, u)]$ be a set of representatives of orbits. For $(P_\gamma, \pi) \in \mathcal{L}_b(G, L, u)$, we set

$$Aut(L, u)_{\overline{(P_\gamma,\pi)}} = \{\varphi \in Aut(L, u) \mid \exists g \in N_G(P_\gamma),\ \pi\varphi\pi^{-1} = Res(i_g)\}.$$

Theorem

Multiplicities (3)

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Theorem

The multiplicity of $S_{L,u,V}$ in $\mathbb{F}T^\Delta_{G,b}$

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Theorem

The multiplicity of $S_{L,u,V}$ in $\mathbb{F}T^{\Delta}_{G,b}$ is the dimension of

$$\bigoplus_{(P_\gamma,\pi)\in[\mathcal{L}_b(G,L,u)]} V^{Aut(L,u)_{\overline{(P_\gamma,\pi)}}}.$$

←