The Green biset functor of complex characters

Serge Bouc

CNRS-LAMFA
Université de Picardie

MFO-2609, February 22-27, 2026

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- Tensor product is left adjoint to internal hom.

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Opposite GBF - Tensor product of GBF's

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- *For a finite group* G*, and* $s \in \hat{\mathbb{Z}}^\times$*, let* $\Psi_G^s : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(G)$ *be defined by* $\Psi_G^s(\chi)(g) := \chi(g^{\tilde{s}_n})$ *for* $g \in G$

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For Assertion 2, reduce to cyclic groups, using Brauer's induction theorem.

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(A map $f : \hat{\mathbb{Z}}^\times \to \mathbb{Z}$ is called locally constant if there exists a positive integer n such that f factors as $f : \hat{\mathbb{Z}}^\times \xrightarrow{\pi_n} (\mathbb{Z}/n\mathbb{Z})^\times \xrightarrow{\tilde{f}} \mathbb{Z}$).

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Bar resolution

Bar resolution [Romero - 2024]

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A Green biset functor A is called separable if $\mu_A : A \otimes A \to A$ *is a split epimorphism of* (A, A)*-bimodules.*

Theorem (Romero - 2024)

A Green biset functor A is separable if and only if $\mathcal{H}H^1(A, M) = 0$ *for any* (A, A)*-bimodule M.*

Theorem

1. $\mathcal{H}H^1(R_{\mathbb{C}}, R_{\mathbb{C}}) = 0$.
2. *Let* $I := Ker(\mu_{R_{\mathbb{C}}}) : R_{\mathbb{C}} \otimes R_{\mathbb{C}} \to R_{\mathbb{C}}$. *Then* $\mathcal{H}H^1(R_{\mathbb{C}}, I) \neq 0$. *In particular* $R_{\mathbb{C}}$ *is not separable.*

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2. *Let $I := Ker(\mu_{R_\mathbb{C}}) : R_\mathbb{C} \otimes R_\mathbb{C} \to R_\mathbb{C}$. Then $\mathcal{H}H^1(R_\mathbb{C}, I) \neq 0$. In particular $R_\mathbb{C}$ is not separable.*