INVARIANCE OF GENERALISED REYNOLDS IDEALS UNDER DERIVED EQUIVALENCES

ALEXANDER ZIMMERMANN

ABSTRACT. For any algebraically closed field k of positive characteristic p and any non negative integer n Külshammer defined ideals $T_n A^\perp$ of the centre of a symmetric k-algebra A. We show that for derived equivalent algebras A and B there is an isomorphism of the centres of A and B mapping $T_n A^\perp$ to $T_n B^\perp$ for all n. Recently Héthelyi, Horváth, Külshammer and Murray showed that this holds for Morita equivalent algebras.

INTRODUCTION

Let k be an algebraically closed field of characteristic $p > 0$ and let A be a finite dimensional symmetric k-algebra with non degenerate symmetrising bilinear form $(\ ,\ )$ on A. Külshammer defined in [6] ideals $T_n A^\perp$ of the centre of A by the following construction. Let KA be the k-subspace of A generated by $ab - ba$ for all $a, b \in A$ and set $T_n A := \{x \in A \mid x^{p^n} \in KA\}$. Let $T_n A^\perp$ is the subspace orthogonal to $T_n A$ with respect to the form $(\ ,\ )$ on A. Note that $T_n A^\perp$ is then an ideal of ZA as $ZA = KA^\perp$ and that $T_n A$ is a ZA submodule of A

In [7] Külshammer shows that the equation $(\zeta_n(z), x)^{p^n} = (z, x^{p^n})$ for any x, z in the centre of A defines a mapping ζ_n from the centre of A to the centre of A. Moreover, $\zeta_n(A) = T_n A^\perp$. Many properties of group algebras can be shown using the ideals $T_n A^\perp$. Concerning the ideals $T_n A^\perp$, Héthelyi et al. show in [2] that $Z_0 A \subseteq (T_1 A^\perp)^2 \subseteq HA$, where HA is the Higman ideal of A, and where $Z_0 A$ is the sum of the centres of those blocks of A which are simple algebras. They show that for odd p the left inclusion is an equality, whereas for $p = 2$ one gets $Z_0 A = (T_1 A^\perp)^3 = (T_1 A^\perp) \cdot (T_2 A^\perp)$. Finally, the authors show that $e \cdot (T_n A^\perp) \cdot e = T_n(eAe)^\perp$ for any idempotent e of A. Now, the authors use the fact that within all algebras Morita equivalent to A there is an, up to isomorphism unique, smallest algebra $B = eAe$ Morita equivalent to A, the basic algebra. If A is symmetric, B is symmetric as well, as follows in a more general context by [11]. Multiplication by this idempotent induces an isomorphism between the centers of an algebra A and its basic algebra B. Hence, the corresponding ideals $T_n A^\perp$ and $T_n B^\perp$ are sent to each other by this isomorphism. Composing two of them gives a corresponding statement for Morita equivalent algebras.

In [2, question 5.4] Héthelyi et al. ask whether for two symmetric algebras A and B, the condition that the derived categories of A and of B are equivalent imply the existence of an isomorphism φ of their centres so that φ induces an isomorphism between the ideals $T_n A^\perp$ and $T_n B^\perp$ for all $n \in \mathbb{N}$. The main objective of this paper is to give a positive answer to this question.

This way we provide new invariants for an equivalences between the triangulated categories $D^b(A)$ and $D^b(B)$ for algebras A and B. A number of invariants are known. Suppose $D^b(A) \simeq D^b(B)$ as triangulated categories, then we get an isomorphism of the Hochschild homology $HH_*(A) \simeq HH_*(B)$ and the Hochschild cohomology $HH^*(A) \simeq HH^*(B)$ (cf Rickard [9]), the cyclic homology $HC_*(A) \simeq HC_*(B)$, the cyclic cohomology $HC^*(A) \simeq HC^*(B)$ of the algebra A (Keller [4]), or by a result of Thomason and Trobaugh the K-theory $K_*(A) \simeq K_*(B)$. Some of them are quite useful in specializing the degree. So is $HH^0(A) \simeq Z(A)$ the centre of the algebra A, or $rank_\mathbb{Z}(K_0(A))$ equals the number of isomorphism classes of simple A-modules. Nevertheless, if these few computable invariants coincide, it is in general very difficult to decide

whether two algebras have equivalent derived categories or not. So, invariants which are more easy to determine in examples will be very welcome. Our result provides some of them.

The main result Theorem 1 will be proven in Section 3. Since there is no analogue of a basic algebra for derived equivalences, we need to proceed differently from Héthelyi's et al.'s proof for Morita equivalence. Section 1 recalls some of the relevant notation and results from homological algebra, for the convenience of the reader. We use the characterisation [7, (46)]; or [2, Lemma 2.1] of $T_n A^\perp$ as the image of the mapping ζ_n^A and define in Section 2 the mapping ζ_n^A in a functorial manner by means of a composition of mappings between $A \otimes_k A^{op}$-modules. We apply the derived equivalence to each of the factors and using results in [11], and some delicate commutativity considerations we are able to show that the mapping induced by a standard derived equivalence on the morphism sets are indeed as asked. For notations concerning derived categories and equivalences we follow [5]. Other references covering the needed background are for example Gelfand-Manin [1] or Weibel [10]. The notation may differ slightly there.

1. A crash course on the relevant homological algebra

For the reader's convenience and to fix notation we shall recall some basic facts in homological algebra as it is needed in the sequel. Basic source is the book [5], and for some more general aspects Gelfand-Manin [1], Weibel [10], or Rickard [9] as well as [11].

For a commutative noetherian ring k and a finitely generated k-algebra A we denote the category of finitely generated left A-modules by $A - mod$ and the category of all A-modules by $A - Mod$. Let $K(A - mod)$ be the category of complexes in $A - mod$ modulo homotopy. Recall that the derived category $D^b(A)$ of bounded complexes of finitely generated A-modules is formed by bounded complexes in $K(A - mod)$ and formally inverting morphisms which induce isomorphisms on homology. Recall furthermore that $A - mod$ is a full subcategory of $D^b(A)$ by mapping a module M to a complex with homogeneous components 0 in all degrees except in degree 0 where the homogeneous component is M (cf e.g. Gelfand-Manin [1, III §5 Proposition 2]). Hence, any two objects M and N of A-mod may be considered as object in $D^b(A)$, and then $Hom_{D^b(A)}(M, N) = Hom_A(M, N)$. This fact will be used at various places.

Recall that in case X is a complex in $D^b(A)$ whose homogeneous components are all projective, then for any complex Y in $D^b(A^{op})$ one has $Y \otimes_A X = Y \otimes_A^{\mathbb{L}} X$. In case A and B are two algebras over a field k, then for any X in $D^b(A \otimes_k B^{op})$ there is an $\tilde{X}$ in $D^b(A \otimes_k B^{op})$ so that $X \simeq \tilde{X}$ and so that all homogeneous components of $\tilde{X}$ are projective as A-modules and as B^{op}-modules (cf [5, Lemma 6.3.12]).

Let B be a k-algebra which is projective as k-module. By a result due to Keller (cf e.g. [3]) or [5, Chapter 8]) $D^b(A)$ is equivalent to $D^b(B)$ as triangulated categories if and only if there is a complex X in $D^b(B \otimes_k A)$ so that $X \otimes_A^{\mathbb{L}} - : D^b(A) \longrightarrow D^b(B)$ is an equivalence. Such equivalences are called standard and X is called (two-sided) tilting complex. If B is symmetric, then A is symmetric as well (cf [11]) and then the inverse equivalence to $X \otimes_A^{\mathbb{L}} -$ is given by $Hom_k(X, k) \otimes_B^{\mathbb{L}} -$. Moreover, Rickard has shown that this $X \otimes_A^{\mathbb{L}} - \otimes_A^{\mathbb{L}} Hom_k(X, k)$ actually defines an equivalence $D^b(A \otimes_k A^{op}) \longrightarrow D^b(B \otimes_k B^{op})$ where the $(A \otimes_k A^{op})$-module A is mapped to B (cf Rickard [9]; or [5, Proposition 6.2.6]). Hence X induces an isomorphism

$$Z(A) = End_{A \otimes_k A^{op}}(A) \quad = End_{D^b(A \otimes_k A^{op})}(A) \quad \simeq$$
$$End_{D^b(B \otimes_k B^{op})}(B) = \quad End_{B \otimes_k B^{op}}(B) = Z(B).$$

This isomorphism is explicitly exhibited in [5, Proposition 6.2.6]. In [11] it is shown that under the equivalence induced by tensoring with X the $(A \otimes_k A^{op})$-module $Hom_k(A, k)$ is mapped to $Hom_k(B, k)$.

We finish with some notation. Let $\mathcal{C}$ be a category and let X, Y and Z be any three objects in $\mathcal{C}$. We denote for any morphism $\varphi \in Hom_{\mathcal{C}}(X, Y)$ the induced mapping $Hom_{\mathcal{C}}(Z, \varphi)$: $Hom_{\mathcal{C}}(Z, X) \longrightarrow Hom_{\mathcal{C}}(Z, Y)$ which is defined by $(Hom_{\mathcal{C}}(Z, \varphi))(\psi) := \varphi \circ \psi$ for any $\psi \in Hom_{\mathcal{C}}(Z, X)$. If Z is clear from the context, we write $Hom_{\mathcal{C}}(Z, \varphi) =: \varphi_*$ for short.

2. INTERPRETING ζ

Recall from Section 1 that $Hom_{D^b(A \otimes_k A^{op})}(A, A) = End_{A \otimes_k A^{op}}(A) \simeq Z(A)$.

Furthermore, by the adjointness formulas (cf e.g. Mac Lane [8, VI (8.7)]), we get

$$Hom_k(A \otimes_{A \otimes_k A^{op}} A, k) \quad \simeq \quad Hom_{A \otimes_k A^{op}}(A, Hom_k(A, k))$$
$$f \quad \mapsto \quad (a \mapsto (b \mapsto f(a \otimes b)))$$

and since canonically by the very definition of a tensor product $A \otimes_{A \otimes_k A^{op}} A \simeq A/KA$ where $KA = \sum_{a,b \in A} k \cdot (ab - ba)$ is the k-vector space generated by commutators, we have a functorial isomorphism

$$Hom_k(A/KA, k) \quad \simeq \quad Hom_{A \otimes_k A^{op}}(A, Hom_k(A, k))$$
$$f \quad \mapsto \quad (a \mapsto (b \mapsto f(ab)))$$

The mapping $A/KA \ni a \mapsto a^p \in A/KA$ was first defined by Richard Brauer who called it the Frobenius mapping and proved that it is well defined (cf Külshammer [6, II]) and semilinear. Denote by $k^{(n)}$ the n times Frobenius twisted copy of k.

The Frobenius mapping induces a well defined mapping

$$Hom_k(A/KA, k) \quad \longrightarrow \quad Hom_k(A/KA, k^{(1)})$$
$$f \quad \mapsto \quad (a \mapsto f(a^p))$$

The mapping

$$Fr^k_* : \ Hom_k(A/KA, k) \quad \longrightarrow \quad Hom_k(A/KA, k^{(1)})$$
$$f \quad \mapsto \quad (a \mapsto f(a)^p)$$

induces a mapping

$$Hom_{A \otimes_k A^{op}}(A, Hom_k(A, k)) \longrightarrow Hom_{A \otimes_k A^{op}}(A, Hom_k(A, k^{(1)}))$$

and since for any algebra B one has a fully faithful embedding of $B - mod$ into $D^b(B)$ by considering a B-module as a complex with differential 0 and modules concentrated in degree 0 only, this in turn gives a mapping

$$Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k)) \longrightarrow Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k^{(1)})).$$

Put $A^* := Hom_k(A, k)$. Recall that a k-algebra A is symmetric if and only if there is an isomorphism of $A \otimes_k A^{op}$-bimodules $A \simeq A^*$, or equivalently there is a non degenerate symmetric bilinear form $(,) : A \times A \longrightarrow k$ satisfying $(a, cb) = (ac, b)$ for any $a, b, c \in A$ (cf e.g. [5, Chapter 9]). Then the mapping ζ_n^A is defined by the equation $(\zeta_n(z), x)^{p^n} = (z, x^{p^n})$ which can be written as composition of the mappings in the following diagram ($\ddagger$):

$$
\begin{array}{ccc}
Hom_{D^b(A \otimes_k A^{op})}(A, A) & \longrightarrow & Hom_{D^b(A \otimes_k A^{op})}(A, A^*) \\
& & \downarrow ((Fr^k)_*)^n \\
(\ddagger) \qquad \uparrow \zeta_n^A & & Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k^{(n)})) \\
& & \uparrow ((Fr^A)^*)^n \\
Hom_{D^b(A \otimes_k A^{op})}(A, A) & \longrightarrow & Hom_{D^b(A \otimes_k A^{op})}(A, A^*)
\end{array}
$$

where the horizontal arrows are induced by the isomorphism

$$A \quad \longrightarrow \quad A^*$$
$$a \quad \mapsto \quad (b \mapsto (a, b))$$

which is coming from the symmetrising bilinear form $(,) : A \otimes_k A \longrightarrow k$. of A.

3. Behaviour under derived equivalences

In this section we prove our main result.

Theorem 1. *Let k be an algebraically closed field of characteristic $p > 0$ and let A and B be finite dimensional k-algebras. If $D^b(A) \simeq D^b(B)$ as triangulated categories, then there is an isomorphism $\varphi : ZA \longrightarrow ZB$ between the centres ZA of A and ZB of B so that $\varphi(T_nA^{\perp}) = T_nB^{\perp}$ for all positive integers $n \in \mathbb{Z}$.*

Remark 3.1. This answers to the positive question 5.4 posed by László Héthelyi, Erszébet Horváth, Burkhard Külshammer and John Murray in [2].

Proof: Let $F : D^b(A) \longrightarrow D^b(B)$ be a standard derived equivalence with two-sided tilting complex X. Let X' be the inverse tilting complex. Then, in [11] it is shown that $X \otimes_A - \otimes_A X'$ induces an equivalence $G : D^b(A \otimes_k A^{op}) \longrightarrow D^b(B \otimes_k B^{op})$ mapping the A-A-bimodule $_AA_A$ to the B-B-bimodule $_BB_B$,

$$G(\,_AA_A) =\ _BB_B.$$

From [11, Lemma 1] we know that

$$G\left(Hom_k(A, k)\right) = Hom_k(B, k).$$

We shall show that

$$G\left(Hom_k(A, k^{(n)})\right) = Hom_k(B, k^{(n)})$$

for all $n \in \mathbb{Z}$. Indeed, $X \otimes_A - \simeq Hom_A(X', -)$ and $- \otimes_A X' \simeq Hom_A(X, -)$ by the adjointness properties of Hom and $\otimes$-functors. Hence (cf [5, proof of Corollary 6.3.6]),

$$
\begin{aligned}
X \otimes_A Hom_k(A, k^{(n)}) \otimes_A X' &\simeq Hom_A(X', Hom_k(A, k^{(n)})) \otimes_A X' \\
&\simeq Hom_k(A \otimes_A X', k^{(n)}) \otimes_A X' \\
&\simeq Hom_k(X', k^{(n)}) \otimes_A X' \\
(\dagger) \qquad &\simeq Hom_A(X, Hom_k(X', k^{(n)})) \\
&\simeq Hom_k(X \otimes_A X', k^{(n)}) \\
&\simeq Hom_k(B, k^{(n)})
\end{aligned}
$$

We apply now G to the diagram $(\ddagger)$ of Section 2 and get a commutative diagram

$$
\begin{array}{ccc}
Hom_{D^b(B \otimes_k B^{op})}(B, B) & \longrightarrow & Hom_{D^b(B \otimes_k B^{op})}(B, B^*) \\
G \uparrow \simeq & & G \uparrow \simeq \\
Hom_{D^b(A \otimes_k A^{op})}(A, A) & \longrightarrow & Hom_{D^b(A \otimes_k A^{op})}(A, A^*) \\
& & \downarrow ((Fr^k)_*)^n \\
\uparrow \zeta_n^A & & Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k^{(n)})) \\
& & \uparrow ((Fr^A)^*)^n \\
Hom_{D^b(A \otimes_k A^{op})}(A, A) & \longrightarrow & Hom_{D^b(A \otimes_k A^{op})}(A, A^*) \\
G \downarrow \simeq & & G \downarrow \simeq \\
Hom_{D^b(B \otimes_k B^{op})}(B, B) & \longrightarrow & Hom_{D^b(B \otimes_k B^{op})}(B, B^*)\ .
\end{array}
$$

It is clear that the upper and the lower square are commutative, since they arise as squares induced from applying an equivalence of categories.

Recall the notation we use as explained at the end of Section 1.

We obtain a commutative diagram

$$
\begin{array}{ccc}
Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k)) & \xrightarrow{\ G\ } & Hom_{D^b(B \otimes_k B^{op})}(B, Hom_k(B, k)) \\
\downarrow Hom_k(A, (Fr^k)^n) & & \downarrow \varphi \\
Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k^{(n)})) & \xrightarrow{\ G\ } & Hom_{D^b(B \otimes_k B^{op})}(B, Hom_k(B, k^{(n)}))
\end{array}
$$

where $\varphi = G \circ Hom_k(A, (Fr^k)^n) \circ G^{-1}$.

We shall need to see that $\varphi = Hom_k(B, (Fr^k)^n)$.

Claim 1. $G \circ Hom(A, Fr^k) = Hom(B, Fr^k) \circ G$.

Proof: Observe that $G = X \otimes_A - \otimes_A X'$ acts only on the contravariant variables. Going through the isomorphisms (†), since Fr^k acts on the covariant variable only, this proves the claim. ∎

Therefore, the diagram

$$
\begin{array}{ccc}
Hom_{D^b(B\otimes_k B^{op})}(B, B^*) & \overset{Hom(B,(Fr^k)^n)}{\longrightarrow} & Hom_{D^b(B\otimes_k B^{op})}(B, Hom_k(B, k^{(n)})) \\
\simeq\uparrow G & & \simeq\uparrow G \\
Hom_{D^b(A\otimes_k A^{op})}(A, A^*) & \overset{Hom(A,(Fr^k)^n)}{\longrightarrow} & Hom_{D^b(A\otimes_k A^{op})}(A, Hom_k(A, k^{(n)}))
\end{array}
$$

is commutative and the vertical morphisms are isomorphisms since G is an equivalence, and since the images of the various objects under G in their version A and B correspond to each other.

Claim 2. $G \circ Hom(Fr^A, k) = Hom(Fr^B, k) \circ G$.

Before starting with the proof observe the following consequences. Once the claim is established the diagram

$$
\begin{array}{ccc}
Hom_{D^b(B\otimes_k B^{op})}(B, B^*) & \overset{Hom((Fr^B)^n,k)}{\longrightarrow} & Hom_{D^b(B\otimes_k B^{op})}(B, Hom_k(B, k^{(n)})) \\
\simeq\uparrow G & & \simeq\uparrow G \\
Hom_{D^b(A\otimes_k A^{op})}(A, A^*) & \overset{Hom((Fr^A)^n,k)}{\longrightarrow} & Hom_{D^b(A\otimes_k A^{op})}(A, Hom_k(A, k^{(n)}))
\end{array}
$$

is commutative.

Observe that since $Hom_k(Fr^A, k)$ is not $A \otimes_k A^{op}$-linear, the functor G is not defined on $Hom_k(Fr^A, k)$. Hence, the only way to prove the commutativity of the above diagram is by inspection of the values.

Proof of Claim 2: We need to make explicit the mappings

$$G : Hom_{D^b(A\otimes_k A^{op})}(A, A^*) \longrightarrow Hom_{D^b(B\otimes_k B^{op})}(B, B^*)$$

and

$$G : Hom_{D^b(A\otimes_k A^{op})}(A, Hom_k(A, k^{(1)})) \longrightarrow Hom_{D^b(B\otimes_k B^{op})}(B, Hom_k(B, k^{(1)})).$$

For this, it is useful, and possible, to replace B by $X \otimes_A X'$ and A by $X' \otimes_B X$.

We first deal with the first identification. Then, again by the usual adjointness formula between Hom and $\otimes$, one has to make explicit an isomorphism

$$G : Hom_k(A \otimes_{A\otimes_k A^{op}} A, k) \longrightarrow Hom_k(B \otimes_{B\otimes_k B^{op}} B, k),$$

or, replacing B by $X \otimes_A X'$ and A by $X' \otimes_B X$,

$$G : Hom_k((X' \otimes_B X) \otimes_{A\otimes_k A^{op}} (X' \otimes_B X), k) \to Hom_k((X \otimes_A X') \otimes_{B\otimes_k B^{op}} (X \otimes_A X'), k).$$

The isomorphism

$$A \otimes_{A\otimes_k A^{op}} A \simeq A/KA \simeq B/KB \simeq B \otimes_{B\otimes_k B^{op}} B$$

comes from a mapping

$$(x \otimes y) \otimes 1_A \mapsto (y \otimes x) \otimes 1_B$$

where it is clear that this is well defined. The diagonal mapping

$$
A \longrightarrow \underbrace{A \otimes_A A \otimes_A \cdots \otimes_A A}_{p \text{ factors}}
$$
$$
a \mapsto a \otimes a \otimes \cdots \otimes a
$$

is exactly the p-power map $A \ni a \mapsto a^p \in A/KA$. If k is of characteristic p, then this is additive and factors through $A \longrightarrow A/KA$.

Now, we observe that A/KA is equally isomorphic to

$$\underbrace{A \otimes_A A \otimes_A \cdots \otimes_A A}_{p-1 \text{ factors}} \otimes_{A\otimes_k A^{op}} A.$$

Moreover, seen $A \simeq X' \otimes_B X$ and $B \simeq X \otimes_A X'$ and one recovers an isomorphism

$$(X' \otimes_B X)^{\otimes_A{}^{p-1}} \otimes_{A \otimes_k A^{op}} (X' \otimes_B X) \longrightarrow (X \otimes_A X')^{\otimes_B{}^{p-1}} \otimes_{B \otimes_k B^{op}} (X \otimes_A X')$$

$$(x_i \otimes y_i)_{i=1}^{p-1} \otimes (x_p \otimes y_p) \mapsto ((y_p \otimes x_1) \otimes (y_i \otimes x_{i+1})_{i=1}^{p-2}) \otimes (y_{p-1} \otimes x_p)$$

which is an incarnation of the isomorphism $A/KA \longrightarrow B/KB$. We need to show that this is well-defined, but actually this is just a straight forward and detailed examination which ring acts where in which way.

Therefore, the p-power map on A/KA is mapped to the p-power map on B/KB by a standard derived equivalence.

We need to explain the second isomorphism

$$G : Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k^{(1)})) \longrightarrow Hom_{D^b(B \otimes_k B^{op})}(B, Hom_k(B, k^{(1)})).$$

Here, we observe that

$$Hom_{D^b(A \otimes_k A^{op})}(A, Hom_k(A, k^{(1)})) \simeq Hom_k(A \otimes_{A \otimes_k A^{op}} A, k^{(1)})$$

and the very same arguments and constructions as above hold. The only difference is that one needs to consider semilinear mappings only at the end. The reorganization procedure is just the same. In particular, the action of Fr^B consists in tensoring the whole term on the right p times over $B \otimes B^{op}$. It is now immediate to see that this operation commutes with this reorganization of factors as described by explaining ν. So,

$$G \circ Hom(Fr^A, k) = Hom(Fr^B, k) \circ G.$$

∎

Claim 3. *The images of $G \circ \zeta_n^A \circ G^{-1}$ and of ζ_n^B coincide.*

Proof: Since $\varphi : A \longrightarrow Hom_k(A, k)$ is an isomorphism of $A \otimes_k A^{op}$-modules, and since G is a functor, $G\varphi$ is an isomorphism as well. As we know that choosing an isomorphism $B \longrightarrow Hom_k(B, k)$ is equivalent to choosing a symmetrising form making B into a symmetric algebra, we may well work with this form instead of the original one. Actually, given two different isomorphisms $\phi : B \longrightarrow Hom_k(B, k)$ and $\psi : B \longrightarrow Hom_k(B, k)$, then for all $x \in B$ one has $\phi^{-1}\psi(x) = \lambda x$ for an invertible central $\lambda \in Z(B)^*$. So, the resulting ζ_n^B differ by invertible central elements. As a consequence, the images are identical. ∎

We shall finish the proof of the theorem. By the previous claims the composition

$$ZB \xrightarrow{G\phi} Hom_k(B/KB, k) \xrightarrow{(G(Fr^A)^* G^{-1})^n} Hom_k(B/KB, k^{(n)})$$

$$\xrightarrow{((G(Fr^k)^*)^n)^{-1}} Hom_k(B/KB, k) \xrightarrow{G\phi^{-1}} ZB$$

is a mapping which differs from ζ_n^B by some central unit of B and therefore the isomorphism induced by G between the centres of A and B maps $T_n A^\perp$ to $T_n B^\perp$.

This finishes the proof of the theorem. ∎

References

[1] Sergei Gelfand and Yuri Manin, Methods of homological algebra, Springer Verlag; Berlin-Heidelberg 1996.

[2] László Héthelyi, Erszébet Horváth, Burkhard Külshammer and John Murray, *Central ideals and Cartan invariants of symmetric algebras*, to appear in J. Alg.

[3] Bernhard Keller, *A remark on tilting theory and DG-algebras*, manuscripta math. **79** (1993) 247-253.

[4] Bernhard Keller, *Invariance and licalization for cyclic homology of DG-algebras*, J. of pure and appl. Alg. **123** (1998) 223-273.

[5] Steffen König and Alexander Zimmermann, Derived Equivalences for Group Rings; Lecture Notes in Mathematics 1685, Springer Verlag Berlin-Heidelberg 1998.

[6] Burkhard Külshammer, *Bemerkungen über die Gruppenalgebra als symmetrische Algebra I, II, III and IV*, J. Alg. **72** (1981) 1-7; **75** (1982) 59-69; **88** (1984) 279-291; **93** (1985) 310-323

[7] Burkhard Külshammer, *Group-theoretical descriptions of ring theoretical invariants of group algebras*, Progress in Mathematics **95** (1991) 425-441.

[8] Saunders Mac Lane, Homology, Springer Verlag Berlin-Heidelberg 1975.

[9] Jeremy Rickard, *Derived equivalences as derived functors*, J. London Math. Soc. **43** (1991), 37–48.

[10] Charles Weibel, Introduction to homological algebra, Cambridge university press 1994.

[11] Alexander Zimmermann, *Tilted symmetric orders are symmetric orders*, Arch. Math. **73** (1999) 15-17.

Université de Picardie,
Faculté de Mathématiques et LAMFA (UMR 6140 du CNRS),
33 rue St Leu,
F-80039 Amiens Cedex 1,
France
E-mail address: alexander.zimmermann@u-picardie.fr