Outer group automorphisms may become inner in the integral group ring

Klaus W. Roggenkamp and Alexander Zimmermann

Abstract

In this note we shall construct a finite group G which has an automorphism α, which is not inner; however, the induced automorphism on SG is inner, where S is the ring of algebraic intgers in a suitably chosen algebraic number field. A consequence of our arguments is, that α is inner in KG for every field K.

1 Introduction

It follows from from a result of Coleman [1] that the natural map

$$\Phi : Out(G) \longrightarrow Out(SG)$$

is injective for p–groups. Here S is the ring of integers in a global or a local number field K, in which p is not invertible. It is a question of Jackowski and Marciniak ([4, 3.7]) whether this happens for all finite groups G and coefficient ring $\mathbb{Z} = S$.

However, there is not yet known an example of a finite group G, where this map is not injective for the algebraic integers of a global field K of characteristic 0. If one wants to construct $\alpha \in Aut(G)$ with $\Phi(\alpha)$ inner on SG, then

1. α must be the identity on the conjugacy classes of G,

2. α must be inner in G on the Sylow p–subgroups, combining Sylow theorems and the result of Coleman (cf. [1] and [7, I.II § 2]).

To construct such an α is is a purely group theoretical problem. However, in order to show that α becomes inner on SG we have to use the following ingredients from integral representation theory:

1. We show that α is inner on $\mathbb{Z}G$ semilocally. From this one cannot automatically conclude that α is inner on $\mathbb{Z}G$. The obstruction is an element in the class group of $\mathbb{Z}G$. We use class field theory – this is where K enters – to kill this obstruction.

2. The passage from the local to the semilocal situation becomes possible by interpreting automorphisms as invertible bimodules and using Fröhlich's exact sequence of Picard groups [3].

3. Finally we are in the local respectively complete situation. Here we use Clifford theory [1] to show, that α acts as inner automorphism on the inertia groups, after having applied the theorem of Noether Deuring, to pass to a splitting field. The key point in our construction is to involve quaternion groups in order to keep the inertia groups small.

4. Let now K be a field, then there exists a rational prime p, such that $KG \simeq K \otimes_{\mathbb{Z}_p} \mathbb{Z}_p G$. Thus α induces an inner automorphism on KG.

2 A short review of Clifford theory

Since Clifford theory is essential to our arguments we give a brief summary of it as developed in [7] as far as it is needed here.

1. Let N be a normal complemented subgroup of a finite group G with $(|G/N|, |N|) = 1$.

2. Let, furthermore, R be a complete local Dedekind domain of characteristic 0 with residue field k of characteristic $p > 0$ such that the quotient field K of R is a splitting field for G and all of its subgroups.

3. We assume that p does not divide the order of N.

[1]The first author has learnt the way of using Clifford theory integrally from Leonard Scott.

4. We denote by $I(M)$ the inertia group of an irreducible RN-lattice M in G.

5. We assume that $I(M)$ is normal in G.

6. Let α be a group automorphism of RG that becomes inner in KG.

7. Let e_M be the central primitive idempotent of RN acting as the identity on M. For each t in $I(M)$, the conjugate $t \cdot e_M \cdot t^{-1}$ is again a central primitive idempotent of RN which acts as 1 on M, hence, e_M is centralized by $I(M)$.

8. If α acts as the identity on N, we have $\alpha(e_M) = e_M$.

9. If now $I(M) < G$, then let $g_1, ..., g_s$ be a left transversal of $I(M)$ in G and let $e_i := g_i \cdot e_M \cdot g_i^{-1}; i = 1, ..., s$ be the conjugates of e_M in RG. Since the $\{g_i\}$ form a transversal with respect to the inertia group of M, the conjugates e_i are pairwise orthogonal idempotents in RG. However, $\sum_{i=1}^{s} e_i =: e$ is a central idempotent in RG. Now $RG \cdot e$ is isomorphic to an $(s \times s)$-matrix ring, whose (i, j)-entry equals $B_{i,j} := e_i \cdot RG \cdot e_j$ for $1 \leq i, j \leq s$, the usual Pierce decomposition.

10. If, furthermore, $x^{-1} \cdot \alpha(x) \in I(M)$ for all $x \in G$, then

$$\alpha(e_i) = \alpha(g_i \cdot e_M \cdot g_i^{-1}) = \alpha(g_i) \cdot \alpha(e_M) \cdot \alpha(g_i)^{-1} = e_i$$

For the rest of the section we assume that 1.– 10. hold.

We look at $B_{i,j}$ more closely:

An element $g \in G$ can be written as $n \cdot h$ with $n \in N$ and $h \in G/N$. Then, $h = g_k \cdot t$ with $t \in I(M)$ and a certain k. We denote by χ_i the character of N afforded by e_i. Using the normality of $I(M)$ we calculate

$$\begin{aligned} e_i \cdot n \cdot h \cdot e_j &= \chi_i(n) \cdot e_i \cdot g_k \cdot t \cdot e_j \\ &= \chi_i(n) \cdot g_i \cdot e_M \cdot g_i^{-1} \cdot g_k \cdot g_j \cdot e_M \cdot g_j^{-1} \cdot t. \end{aligned}$$

If $g_k \cdot g_j \notin g_i \cdot I(M)$, which is equivalent to $g_k \notin g_i \cdot g_j^{-1} \cdot I(M)$, again by the normality of $I(M)$, then the above equals 0. Hence,

$$B_{i,i} = g_i \cdot e_M RI(M) \cdot e_M \cdot g_i^{-1}.$$

More precisely, the above calculations show

$$e_i \cdot RG \cdot e_j = g_i \cdot e_M \cdot RI(M) \cdot e_M \cdot g_j^{-1}\,.$$

This allows us to describe explicitely the action of α on $B_{i,j}$:

$$\alpha : g_i \cdot e_M \cdot x \cdot e_M \cdot g_j^{-1} \longrightarrow \alpha(g_i) \cdot e_M \cdot \alpha(x) \cdot e_M \cdot \alpha(g_j^{-1}) \text{ for } x \in RI(M). \quad (1)$$

A central[2] automorphism α of RG fixes the $B_{i,j}$ as set for all i, j if and only if it is conjugation by

$$\begin{pmatrix} u_1 & 0 & & & 0 \\ 0 & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & 0 \\ 0 & & & 0 & u_s \end{pmatrix}$$

with elements $\{u_i \in (RI(M) \cdot e_M) \cap U(KI(M) \cdot e_M) | i = 1, .., s\}$.[3]

If α acts as the identity on $I(M)$ and if moreover $\alpha(g_i) \in g_i \cdot Z(I(M))$, then it acts as identity on $g_i \cdot e_M \cdot RI(M) \cdot e_M \cdot g_i^{-1}$, and furthermore $u_i \in Z(RI(M) \cdot e_M)$ for all $i = 1, ..., s$. Since R is local,

$$Outcent(RI(M) \cdot e_M) \simeq Outcent(Mat_n(RI(M) \cdot e_M)).$$

Consequently, there is an inner automorphism γ of $Mat_n(RI(M) \cdot e_M) = RG \cdot e$ such that

$$\alpha = \gamma \circ \delta$$

with a central automorphism δ acting via the identification

$$B_{i,j} = e_i \cdot RG \cdot e_j = g_i \cdot e_M \cdot RI(M) \cdot e_M \cdot g_j^{-1}$$

on each of the entries $B_{i,j}$. Since δ is then just conjugation by

$$v \in (RI(M) \cdot e_M) \cap U(KI(M) \cdot e_M),$$

[2]An automorphism of a ring acting as the identity on the centre of the ring is called central.

[3]Note that this can always be achieved by multiplication with a central element.

the unit γ has to have the diagonal form analogous to that of α. Hence, there is an element $v \in (RI(M) \cdot e_M) \cap U(KI(M) \cdot e_M)$ and elements $\gamma_i \in U(RI(M) \cdot e_M)$ such that

$$u_i = \gamma_i \cdot v \in Z(RI(M) \cdot e_M).$$

Therefore,

$$u_i \cdot u_j^{-1} \in U(RI(M) \cdot e_M) \text{ for all } 1 \leq i, j \leq s.$$

We summarize these observations, which we shall apply in the next section, as

Proposition 1 *With the above notation, assume that one of the elements u_i can be chosen to be a unit in $RI(M) \cdot e_M$, then the automorphism α is inner.*

3 The construction of the group and the automorphism

Let $\overline{H}$ be the semidirect product of the cyclic group of order 8, generated by a, with its automorphism group $C_2 \times C_2$, generated by $\overline{b}$ and $\overline{c}$ such that $\overline{b}$ inverts a and $\overline{c}$ raises a to the third power. $\overline{H}$ has a non inner central automorphism σ sending $\overline{c}$ to $a^4\overline{c}$ and fixing $\overline{b}$ and a (cf. [4]). Let

$$H_0 := (C_8 \rtimes (C_2 \times C_2)) \rtimes < \sigma > .$$

Factoring the centre of a quaternion group Q_8 of order 8, the group Q_8 generated by b and c maps onto $C_2 \times C_2$, the image of b being $\overline{b}$ and that of c being $\overline{c}$, the natural projection is called π_Q. We form the pullback

$$\begin{array}{ccccccccc}
1 & \longrightarrow & C_8 & \longrightarrow & H_0 & \longrightarrow & C_2 \times C_2 \times < \sigma > & \longrightarrow & 1 \\
 & & \| & & \uparrow & & \uparrow \pi_Q \times id_{<\sigma>} & & \\
1 & \longrightarrow & C_8 & \longrightarrow & H & \longrightarrow & Q_8 \times C_2 & \longrightarrow & 1.
\end{array}$$

For H we form two irreducible modules M_1 and M_2 such that for each $i \in \{1, 2\}$ the element a acts as 1 on M_i and restricted to the quaternion group Q_8 the modules M_i are the two dimensional irreducible representation of Q_8 over a finite prime field $\mathbb{F}_i$ of odd characteristic p_i. It is well known that there are irreducible two dimensional representations of the quaternion group of order 8 over every field of odd characteristic (see e.g. [2, (27.1) and (74.11)]). We choose $p_1 \neq p_2$. On M_1 the element σ operates as 1 and on M_2 it acts as -1. Let G be the semidirect product of M with H:

$$G := (M_1 \times M_2) \rtimes H$$

Since a acts trivially on $M := M_1 \times M_2$ the automorphism α, conjugation by σ on H, can be extended to G as (id_M, α), also denoted by α. The extension is central since on H the elements h and $\alpha(h)$ are conjugate by either a or a^2 or 1. All of these elements centralize M, and therefore, α as automorphism of G is central. In fact, for every integers i and j the elements $a^ibc\sigma^j$ and $a^ib^3c\sigma^j$ are centralized by $a\sigma$, the elements $a^ic\sigma^j$ and $a^ib^2c\sigma^j$ are centralized by $a^2\sigma$ and the rest of H is centralized by σ itself.

4 The automorphism is semilocally inner

We claim that α is inner in RG with a suitable semilocal ring of integers R, containing a $p_1 \cdot p_2 \cdot 8$-th root of unity, and neither p_1 nor p_2 nor 2 are invertible in R. Note that the Noether Deuring theorem ([6]) implies that there is no loss of generality if we assume that the quotient field of R is a splitting field for G. This can be applied since the outer central automorphism group of RG can be mapped into $Picent(RG)$ (cf. [3]) by means of mapping the automorphism ϕ to the invertible bimodule ${}_{\phi}RG_1$ where the left module action is twisted by ϕ. For details see [3, Theorem 1].

Using Fröhlich's result ([3, Theorem 6]) for semilocal domains R

$$Outcent(RG) \simeq \prod_{\wp \in Spec(R)} Outcent(R_{\wp}G)$$

and interpreting α as an invertible bimodule as above it is enough to show that α is inner for all completions of R at finite primes $\wp$: We extend α to a central automorphism $\alpha_{\wp}$ of $R_{\wp}G$ and map the bimodule corresponding to α to the finite direct product of bimodules corresponding to $\alpha_{\wp}$.

For non semilocal Dedekind domains R the obstruction to globalize local automorphisms is a certain subgroup of the class group of the centre of the group ring. In fact, denoting by $Cl(centre(\Lambda))$ the locally free class group of the order Λ, the sequence

$$1 \longrightarrow Cl(centre(RG)) \longrightarrow Picent(RG) \longrightarrow \prod_{\wp \in Spec(R)} Picent(R_\wp G) \longrightarrow 1$$

Using class field theory (cf. [5, Satz 7]) this implies that there exists a ring of algebraic integers S in an algebraic number field L, L being finite over the quotient field of R, such that α becomes inner as an automorphism of SG.

4.1 The crucial calculation

We now come to the central point in the proof:

We shall show in the next subsection that the following three groups will occur as inertia groups at various primes $\wp$:

1. $I_1 :=< M, a, \sigma >$ is normal in G.
2. $I_2 :=< M, a, \sigma \cdot b^2 >$ is normal in G.
3. $I_3 :=< M, a >= I_1 \cap I_2$.

We take this as guaranteed for the moment. At the prime $\wp$ the group ring has a ring direct factor of the form $(g_i \cdot e_M \cdot R_\wp I \cdot e_M \cdot g_j^{-1})_{1 \leq i,j \leq s}$ with $I \in \{I_1, I_2, I_3\}$. Recall that $e_i = {}^{g_i}e_M$ for $g_i \in H_k$ and H_k suitably chosen according to the inertia groups I_k.

We now consider the three cases I_1 , I_2 and I_3.
In case $I = I_1$ we can choose the transversal

$$H_1 := \{1, b, b^2, b^3, c, cb, cb^2, cb^3\} = H_1^0 \dot\cup c \cdot H_1^0.$$

In case $I = I_2$ we can choose the transversal

$$H_2 := \{1, b, b^2, b^3, c, cb, cb^2, cb^3\} = H_2^0 \dot\cup c \cdot H_2^0.$$

In case $I = I_3$ we can choose the transversal

$$H_3 := \{1, b, b^2, b^3, \sigma, b\sigma, b^2\sigma, b^3\sigma, c, cb, cb^2, cb^3, c\sigma, cb\sigma, cb^2\sigma, cb^3\sigma\} = H_3^0 \dot\cup c{\cdot}H_3^0.$$

We have seen that

$$e_i \cdot R_\wp G \cdot e_j = g_i \cdot e_M \cdot R_\wp I \cdot e_M \cdot g_j^{-1}$$

Thus, by Equation (1), in case $I = I_1$ and in case $I = I_2$ the action of α on $(e_i \cdot R_\wp G \cdot e_j)_{1 \le i,j \le s}$ is multiplication by a^4 if $4 \le |i-j|$ and the identity otherwise, hence, conjugation by the matrix

$$\begin{pmatrix} 1_{4\times 4} & 0 \\ 0 & a^4 \cdot 1_{4\times 4} \end{pmatrix}.$$

Here we denote by $1_{m\times m}$ the m by m unit matrix. Therefore, α acts as inner automorphism on the ring direct factor corresponding to e_M.

In case $I = I_3$ a similar observation yields that α acts as conjugation by

$$\begin{pmatrix} 1_{8\times 8} & 0 \\ 0 & a^4 \cdot 1_{8\times 8} \end{pmatrix}$$

Hence, by Proposition 1, α acts as inner automorphism on the ring direct factor corresponding to e_M as well.[4]

4.2 Determination of the inertia groups

We choose a prime $\wp \in Spec(R)$ and discuss various cases for $\wp$.

Case 1: $2 \in \wp$.

Then the group ring decomposes as

$$R_\wp G = I_{R_\wp}(M)G \prod R_\wp H.$$

On $R_\wp H$ the automorphism α is conjugation by σ . For $I_{R_\wp}(M)G$ we apply Clifford theory. Let χ be an irreducible nontrivial character of M as abelian groups (the quotient field of R is a splitting field for M) with kernel $K \subseteq M_1 \times M_2$.

For χ there are three cases which have to be considered separately:

1. If $M_2 \subset K$ then the inertia group I_χ contains M, σ and a. If $h \in H$ lies in I_χ we observe that the sequence

$$1 \longrightarrow < a, \sigma > \longrightarrow H \longrightarrow Q_8 \longrightarrow 1$$

[4] Note how sensitive this construction is with respect to the group G.

splits. Hence, we may assume that $h \in Q_8$, but there are fixed points on neither M_1 nor M_2, a contradiction. Thus

$$I_\chi = < M, a, \sigma > = I_1,$$

and α is seen to be inner.

2. $M_1 \subset K$ then obviously $I_\chi \supseteq < M, a, b^2 \cdot \sigma >$. Similar arguments show that

$$I_\chi = < M, a, b^2 \cdot \sigma > = I_2.$$

Now, here α is also inner.

3. If neither M_1 nor M_2 belongs completely to K the inertia group

$$I_\chi = < M, a > = I_3$$

is the intersection of the two inertia groups of the previous cases since $M = M_1 \times M_2$ and $(\wp, |M|) = 1$. Again we see that α acts as inner automorphism on the factor corresponding to that character of N.

Our automorphism now acts as inner automorphism on each of the factors and $\alpha_\wp$ is inner in $R_\wp G$ provided $2 \in \wp$.

Case 2: $p_1 \in \wp$.

Then the group ring decomposes to

$$R_\wp G = I_{R_\wp}(M_2)G \prod R_\wp G/M_2.$$

On the second factor the automorphism α is conjugation by σ and for the first factor we take an irreducible non trivial character χ of M_2. Then the inertia group is

$$I_\chi = < M, a, b^2 \cdot \sigma > = I_2$$

as before. As above, $\alpha_\wp$ is inner.

Case 3: $p_2 \in \wp$.

Then the group ring decomposes to

$$R_\wp G = I_{R_\wp}(M_1)G \prod R_\wp G/M_1.$$

On the second factor the automorphism α is conjugation by $b^2 \cdot \sigma$ and for the first factor we take an irreducible non trivial character χ of M_1. Then the inertia group is

$$I_\chi =< M, a, \sigma >= I_1$$

as before. Again, as above, Clifford theory tells us that $\alpha_\wp$ is inner.

Case 4: $|G|R_\wp = R_\wp$

Then the group ring $R_\wp G$ is separable and every central automorphism is inner.

We have shown that α is an inner automorphism of RG.

Now we show that α is not inner as group automorphism. On G/M the automorphism is conjugation by σ. The conjugating elements on G/M are, therefore, contained in $\sigma\cdot < a^4, b^2 >$. Since a^4 is central in G, we may assume that the conjugating element maps onto an element in $\sigma\cdot < b^2 >$. Therefore, the conjugating element in G is $m \cdot b^{2j} \cdot \sigma$ for some $m \in M$ and $j \in \mathbb{Z}$. But on M_1 this operates trivial if and only if j is even and on M_2 it operates trivial if and only if j is odd. Thus it cannot exist and α is not inner in G.[5]

Now, the invertible bimodule [6] ${}_\alpha(RG)_1$ is isomorphic to ${}_1(RG)_1$, and therefore, by [6], we have ${}_\alpha(\mathbb{Z}_\pi G)_1 \simeq {}_1(\mathbb{Z}_\pi G)_1$ with π being any finite set of primes.

Proposition 2 *For the group G of order $2^7 \cdot p_1^2 \cdot p_2^2$ as above the natural map*

$$Outcent(G) \longrightarrow Outcent(\mathbb{Z}_\pi G)$$

is not injective. For $p_1 = 5, p_2 = 3$ the group G has generators and relations[7] $G =< a, b, c, \sigma, x_1, x_2, y_1, y_2 >$ *subject to the relations* $\{a^8 = 1, b^4 = 1, c^4 = 1, (bc)^2 = b^2 = c^2, (\sigma, a) = 1, (\sigma, b) = 1, \ {}^\sigma c = a^4 c, \ {}^b a = a^7, \ {}^c a = a^3, x_1^5 = 1, x_2^5 = 1, y_1^3 = 1, y_2^3 = 1, (a, x_1) = 1, (a, x_2) = 1, (a, y_1) = 1, (a, y_2) = 1, (\sigma, x_1) = 1, (\sigma, x_2) = 1, \ {}^\sigma y_1 = y_1^{-1}, \ {}^\sigma y_2 = y_2^{-1}, \ {}^b x_1 = x_2, \ {}^b x_2 = x_1^{-1}, \ {}^c x_1 = x_1^2, \ {}^c x_2 = x_2^3, \ {}^b y_1 = y_2, {}^b y_2 = y_1^2, \ {}^c y_1 = y_1^2 \cdot y_2, \ {}^c y_2 = y_1 \cdot y_2, (x_1, x_2), (y_1, y_2), (x_1, y_1), (x_2, y_2)\}$

[5]These arguments are also checked by GAP.
[6]We adopt the notation of [3]
[7]The commutator of g and h is denoted by (g, h)

Remark 1 J. Krempa proved that there can at most be an elementary abelian two group in the kernel of the group homomorphism from $Outcent(G)$ to $Outcent(\mathbb{Z}G)$ (cf. [4, 3.2 Theorem]).

Claim 1 *$Outcent(G)$ is of order 2.*

Proof. Let β be a central automorphism of G. The Schur-Zassenhaus theorem assures that we may vary β by conjugation with an element in M such that $H :=< a, b, c, \sigma >$ is fixed under β. Therefore, we assume that β itself fixes H and we may calculate in H modulo M.

Let β be a central automorphism of H. Since the conjugacy class of a consists of the elements $\{a, a^3, a^5, a^7\}$ the normal subgroup $A :=< a >$ of G is fixed under β. Since β is central and elements are conjugate to their image we may assume that $\beta(b) = b$ (if not we modify β by an inner automorphism). The conjugacy class of c consists of the elements $\{c, a^2c, a^4c, a^6c, c^3, a^2c^3, a^4c^3, a^6c^3\}$. If we vary by conjugation by σ and (or) b, centralizing b, we may assume that $\beta(c) \in \{c, a^2c\}$. The conjugacy class of bc is equal to $\{bc, a^4bc\}$, and therefore, c cannot be mapped to a^2c under β if b is fixed. We may assume that b and c are fixed. The conjugacy class of σ is equal to $\{\sigma, a^4\sigma\}$. The conjugacy class of $a\sigma$ is equal to $\{a\sigma, a^7\sigma\}$. Therefore, from $\beta(a) \in \{a^3, a^5\}$ it follows $\beta(\sigma) = a^4\sigma$ and from $\beta(a) \in \{a, a^7\}$ we conclude $\beta(\sigma) = \sigma$.

We, have only 4 candidates left: These are generated by the two automorphisms of order 2: λ_1 and λ_2 with $\lambda_1(a) = a^7$ and the rest of the generators stay fixed and $\lambda_2(a) = a^3, \lambda_2(\sigma) = a^4\sigma$ and the rest of the generators stay fixed. The conjugacy class of abc is equal to $\{abc, a^5bc, a^7b^3c, a^3b^3c\}$. Hence, neither λ_1 nor λ_2 are central automorphisms.

For the automorphism $\lambda_1 \cdot \lambda_2$ we look at the conjugacy class of ay_2 which has length 8. An automorphism has to map y_2 to either y_2 or to y_2^{-1} since the H–modules ${}^{\lambda_1\lambda_2}M$ and M are equal and an automorphism extending $\lambda_1\lambda_2$ differs on M from the identity by an automorphism of M which preserves the orbits of H on M. But all of the elements in $bc \cdot C_H(a)$ moves y_2 to neither y_2 nor to y_2^{-1}. We see that also $\lambda_1\lambda_2$ cannot be central [8].

Assuming β to be the identity on G/M the automorphism is a module endomorphism of M as G/M-module. The endomorphism ring, however, is the prime field on each of M_1 and M_2 and only the element -1 is realizable by

[8] These statements are also checked by GAP.

the operation of a group element. It may hence be realized by the operation of elements in the subgroup $< b^2, \sigma >$ which causes to be α modulo conjugation by b^2.

We conclude: Up to inner automorphisms of G there is only one non trivial central automorphism of G inducing the identity on H.

Acknowledgement: We would like to thank the referee for pointing out a gap in the proof in an earlier version of the paper.

References

[1] *D. Coleman,* On the modular group ring of a p-group, Proc. Amer. Math. Soc. 15 (1964), 511–514.

[2] *C. W. Curtis and I. Reiner,* Methods of Representation Theory Vol. I & II, John Wiley Interscience, New York, 1982 & 1987.

[3] *A. Fröhlich,* The Picard groups of noncommutative rings, in particular of orders, Trans. Amer. Math. Soc. 180 (1973), 1–46.

[4] *S. Jackowski and Z. Marciniak,* Group automorphisms inducing the identity map on cohomology, J. of Pure and Appl. Alg. 44 (1987), 241–250.

[5] *H. Jackobinski,* Über die Geschlechter von Gittern über Ordnungen, J. Reine Angew. Math. 230 (1968), 29–39.

[6] *K. W. Roggenkamp,* An extension of the Noether Deuring theorem, Proc. Amer. Math. Soc. 31 (1972), 423–426.

[7] *K. W. Roggenkamp and M. J. Taylor,* Class Groups and Group Rings, Birkhäuser Verlag, Basel, 1992.

Address of the authors:
Mathematisches Institut B
Universität Stuttgart
Pfaffenwaldring 57
D–70550 Stuttgart
Germany