A NOTE ON SEMI–HEREDITARY ORDERS

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Introduction

In this note we characterize certain types of semi–herediatry orders in a semisimple artinian ring. The structures of such semi–hereditary orders happen to be somewhat similar to those of classical hereditary orders over complete discrete valuation domains investigated in [Bru] and [Hara]. This might lead to a variety of examples to certain problems concerning the necessity on assumptions to be noetherian. Moreover, we are able to give all the twosided ideals of these orders.

Our motivation for the interest in those semi–hereditary orders came from an example we found while we were trying to find applications for the main theorem in [P-R].

Let R_n be the algebraic closure of the $p-$adic integers in the p^n–th cyclotomic extension field $\widehat{\mathbb{Q}}_p[\zeta_n]$ of the $p-$adic numbers, ζ_n denotes a primitive p^n–th root of unity. Let R be the union of all the R_n in an algebraic closure of the p–adic numbers. Let $J(R)$ be the Jacobson radical of R. Observe that $J(R)$ is not finitely generated as R–module. Let

$$\Lambda := \begin{pmatrix} R & R \\ J(R) & R \end{pmatrix}.$$

The two projective indecomposable modules are P_1 and P_2. The complex $P_1 \oplus (P_1 \longrightarrow P_2)$, both copies of P_1 are in degree 0, is a tilting complex[1] with endomorphism ring

$$End_\Lambda(T) \simeq \begin{pmatrix} R & R/J(R) \\ 0 & R/J(R) \end{pmatrix}$$

[1]The fundamental theorem of Rickard [Ric] states as follows: Rings Γ and Λ are derived equivalent if and only if there is a so called tilting complex T. A tilting complex T is a bounded complex of finitely generated projective Λ–modules such that the triangulated category generated by summands of finite sums of T contains all projective indecomposable Λ–modules and such that $Ext^i_\Lambda(T,T) = 0$ for $i \neq 0$ and $End_\Lambda(T) \simeq \Gamma$.

which is generated by 3 elements as an R–module. We remind the reader that J. Rickard proved in [Ric] that if Λ is finitely generated over a noetherian centre then if Γ is derived equivalent to Λ, also Γ is derived equivalent over the same noetherian centre. The analogous statement appears to be false if one drops the assumption that the centre is noetherian. In analogy to the paper [K-Z] we tried to classify all the rings derived equivalent to semi–hereditary orders with centre R contained in the integral closure of the $p-$adic integers in the algebraic closure of the p–adic numbers.

For generalities on triangulated categories we refer to [Hart] and [Ver].

By our structure theorem it turns out that the proof of [K-Z] on the derived equivalences of hereditary orders carries through word by word and so we are able to give the rings which are derived equivalent to those semi–hereditary orders. We also present a whole series of pairs of rings (Λ,Γ) such that Γ and Λ are derived equivalent but Γ is finitely generated over its centre, while Λ is not.

Also it would be interesting to compare the characterization of these semi–hereditary orders with that of Bezout orders in the forthcoming paper [P-R].

The main theorem and its proof

We denote for any ring S by $J(S)$ the Jacobson radical of S.

Theorem 1. Let Λ be a semi–hereditary prime PI–ring with the centre R, which is local, integrally closed and contained in the algebraic closure $\overline{\mathbb{Q}}_p$ of the p–adic numbers. Let K be the field of fractions of R. Then Λ is Morita equivalent to an order which is of the following shape,

$$\begin{pmatrix} \Delta & \dots & \dots & \Delta \\ J(\Delta) & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ J(\Delta) & \dots & J(\Delta) & \Delta \end{pmatrix}_{n\times n}$$

for a maximal R–order Δ in a skew field.

Since K is a subfield of the algebraic closure $\overline{\mathbb{Q}}_p$ of the p–adics, K is the direct limit of the finite extension fields K_α of the p–adics $\mathbb{Q}_p$, partially ordered by inclusion.

We define $R_\alpha :=$ alg. int.(K_α).

Note that $A := K\Lambda$ is simple artinian (cf. [Row]).

Example 1. We cannot expect to obtain a hereditary order. Let R be the ring of algebraic integers in $\overline{\mathbb{Q}}_p$. Then, let

$$\Lambda = \begin{pmatrix} R & R \\ J(R) & R \end{pmatrix} = lim \begin{pmatrix} R_\alpha & R_\alpha \\ J(R_\alpha) & R_\alpha \end{pmatrix}.$$

The limit is taken over the set of finite extensions of $\mathbb{Q}_p$. Since $J(R)$ is not finitely generated over R ([P-R]), Λ is not hereditary.

Essentially for the proof is the following fact developed in [P-R].

Proposition 1. ([P-R]) Let Λ be a prime PI–ring whose centre $R = lim(R_\alpha)$ is local, integrally closed and consists entirely of algebraic integers over $\overline{\mathbb{Q}}_p$. Then Λ is the limit of classical R_α–orders Λ_α for $\alpha \geq \alpha_0$ for some α_0. Following [P-R] Λ is then called a *semi–order.*

The proof can be found in [P-R].

Proof of Theorem 1.

By Proposition 1 the order Λ is the direct limit of the orders Λ_α. First we shall establish

Lemma 1. For all α, the orders Λ_α are hereditary R_α–orders in $A_\alpha := K_\alpha \Lambda_\alpha$.

Proof. We choose a left ideal I_α of Λ_α. Of course, I_α is finitely generated. For each $\beta \geq \alpha$ we set $I_\beta := \Lambda_\beta \cdot I_\alpha$ which is clearly also finitely generated, over Λ_β. The limit $I := lim_{\beta \geq \alpha} I_\beta$ is hence also finitely generated and by the semi–hereditarity of Λ, projective as Λ–lattice. There exists a surjection

$$\pi : \Lambda^m \longrightarrow I$$

which is split by a morphism σ. Now, since I is finitely generated, σ is already defined over δ which may be larger than α. We furthermore observe that

$$I_\delta = \Lambda_\delta I_\alpha = R_\delta \Lambda_\alpha I_\alpha = R_\delta \otimes_{R_\alpha} I_\alpha,$$

I_δ being R_δ–free. Similarly, $\Lambda_\delta = R_\delta \otimes_{R_\alpha} \Lambda_\alpha$.

For some projective Λ_δ–lattice C_δ we have

$$\Lambda_\delta^m \simeq I_\delta \oplus C_\delta$$

and invoking that $R_\delta \otimes_{R_\alpha} -$ is a morphism between the Grothendieck group of projective Λ_α–lattices, $K_0(proj.lat(\Lambda_\alpha))$, to that for Λ_δ, denoted by $K_0(proj.lat(\Lambda_\delta))$, we see that also C_δ is isomorphic to $R_\delta \otimes_{R_\alpha} C_\alpha$ for some projective Λ_α–lattice C_α. We now use the Noether–Deuring theorem as extended in [Rog] to see that

$$C_\alpha \oplus I_\alpha \simeq \Lambda_\alpha^m.$$

We have shown that Λ_α is hereditary and the lemma is proved.

Notation. Pick an arbitrary α. Let $A_\alpha = Mat_{n_\alpha}(D_\alpha)$ for a skewfield D_α and a natural number n_α. Then the unique maximal order in D_α is denoted by Δ_α and $J(\Delta_\alpha)$ is denoted by J_α.

Claim 1. If Λ is basic, also Λ_α is basic for each α.

In fact, if Λ_α has multiplicities greater than 1 in the projective indecomposable summands as left modules then also $\Lambda = R \otimes_{R_\alpha} \Lambda_\alpha$ would have multiplicities in the indecomposable projective summands greater than 1 as left module.

We construct injective systems from the datas. We start with α_0 and look at all $\alpha \geq \alpha_0$.

Claim 2. The triple $(D_{\alpha_0}, \Delta_{\alpha_0}, J_{\alpha_0})$ transforms, tensoring with R_β over R_{α_0} for $\beta \geq \alpha_0$, to the triple $(D_\beta, \Delta_\beta, J_\beta)$.

Proof of Claim 2. We have the standard form of Λ_α being a hereditary order in A_α. Then, $R_\beta \Delta_{\alpha_0}$ is a maximal order in $K_\beta D_\alpha$ and $R_\beta J_\alpha$ is its radical if and only if Λ_β is hereditary ([Rei]). To prove that fact, we take an indecomposable projective module of Λ_β and an indecomposable projective module of $R_\beta \Delta_\alpha$. The endomorphism rings are firstly Δ_β and secondly $R_\beta \Delta_\alpha$. These are hence isomorphic. The fact that Λ_β is hereditary, however, is the statement of Lemma 1.

The set $\{(R_\beta \Delta_{\alpha_0}, R_\beta J_{\alpha_0}, K_\beta D_{\alpha_0})\}_\beta$ forms an injective system. Let (Δ, J, D) be their injective limit. Observe that $lim\ D_\alpha = D$ where $A = K\Lambda$ is a full matrix ring over the skew field D.

Claim 3. Δ is a maximal order in D.

In fact, surely Δ is an order since it is generated by the same elements as Δ_{α_0} over R. Let Γ be an order containing Δ. Then, take an $x \in \Gamma \setminus \Lambda$. x belongs to some $\Lambda_{\alpha'}$. Since Γ is an order, x is integral over R. It satisfies the monic polynomial

$$\sum_{i=0}^{m} r_i x^i \text{ with } r_m = 1 \text{ and } r_i \in R.$$

We take $\alpha \geq \alpha'$ such that all of the r_i belong to R_α. We claim that

$$\Omega_\alpha := \Delta_\alpha + \Delta_\alpha x + \cdots + \Delta_\alpha x^{m-1} \subseteq \Gamma$$

is an order in D_α. Since x is integral this is a subring of D_α generating D_α over K_α. On the other hand, since Δ_α is finitely generated as R_α–module, the same is true for Ω_α. This proves the claim.

Claim 4. $J = J(\Delta)$.

In fact, let $x \in J$. Then $x \in J_\alpha$ for some large enough α. Let $a, b \in \Delta$. Then $a, b, x \in \Delta_\beta$, enlarging α to β if necessary. Since J_β is the radical of Δ_β, $1 - axb$ is invertible in Δ_β, hence also in Δ. This proves that $J \subseteq J(\Delta)$. The same argument yields the other inclusion, hence the claim.

It is now immediate to see that

$$\Lambda = \begin{pmatrix} \Delta & ... & ... & \Delta \\ J(\Delta) & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ J(\Delta) & \dots & J(\Delta) & \Delta \end{pmatrix}.$$

The twosided ideals

The following may be well known, however, the corresponding statement in [Rei] is formulated only for R to be noetherian.

Lemma 2. Let Δ be as above and let furthermore assume that Δ is basic. Then there is a unique maximal left ideal M of Δ and this is also a two sided ideal of Δ.

Proof. Each of the Δ_α is a maximal order in D_α. The limit of the radicals of Δ_α is the radical of Δ, as proved in Claim 4. Let there be two simple Δ–modules S_1 and S_2. Then, take non zero elements x_1 and x_2 from S_1 and S_2 respectively. Both simples are modules over the field $F := R/J(R)$. Surely, $J(R_\alpha)$ is contained in $J(R)$. Hence, F is a field extension of $F_\alpha := R_\alpha/J(R_\alpha)$. Let $S_{i,\alpha} := \Delta_\alpha \cdot x_i$ for $i = 1, 2$. Then, these are simple Δ_α–modules. In fact, any submodule keeps on being a submodule of lower dimension by tensoring with F, F being flat over F_α. Let ϕ_α be an isomorphism from $S_{1,\alpha}$ to $S_{2,\alpha}$. Tensoring with F over F_α this yields an isomorphism from S_1 to S_2. We proved that $J(\Delta)$ is a maximal left ideal of Δ, which asserts the lemma.

Recall that we abbreviated $J(\Delta) = J$.

Theorem 2. Let Λ be a semi–hereditary order with integrally closed and local centre R contained in the algebraic integers of the algebraic closure $\overline{\mathbb{Q}}_p$ of the p–adic numbers. Every twosided ideal I of Λ is of the following form: It equals the set of n by n matrices with entries in the i, j–th position contained in a twosided ideal $I_{i,j}$ of Δ such that there is an element $a(I) \in \Delta$ with $I_{i,j} = a(I) \cdot \Delta$ or $I_{i,j} = a(I) \cdot J$. Moreover, if $I_{i,j} = a(I) \cdot \Delta$, then $I_{k,l} = a(I) \cdot \Delta$ for each $k \geq i$ and $l \geq j$.

If R is a complete discrete valuation ring, it is noehterian and the statement is proved. So let us assume that R is not discrete.

Note that every onesided ideal of Δ is a twosided ideal since this is true for each Δ_α.

We first need to describe the ideals of Δ.

Lemma 3. Every twosided ideal X of Δ, which is not finitely generated, equals $X = u \cdot J \cap \Delta$ for some element u in $\widehat{\Delta}$, where $\widehat{\Delta}$ denotes the $J(\Delta)$–adic completion of Δ.

Remark. Recall that a ring is *left Goldie* if there is no infinite direct sum of non–zero left ideals and it satisfies the ascending chain condition on left annihilators. If u is not a unit, then Δ/X is not left Goldie.

Proof. To make the proof more readable we develop first the special case where $K\Lambda = (K)_n$ with $K = \mathit{lim}\ \mathbb{Q}_p[\zeta_n]$.

Let $I = \sum_{i\in\mathbb{N}} R\cdot u_i$ be the non finitely generated ideal of R. We have a valuation v defined on the whole of K. Let $m := \mathit{inf}\,\{v(u_i) \mid i \in \mathbb{N}\}$ (observe that if there is an $x \in I$ with $m = v(x)$, then I is principal.) So, we take an element $u \in R$ (observe that we are in a complete situation) with $v(u) = m$. Then, since $v(I \cdot u^{-1}) \geq 0$, all of the elements in $I \cdot u^{-1} =: I'$ are integral. We claim that $I' = J$. We have a local ring and therefore $I' \subseteq J$. Hence, we have to prove that $\pi_i \in I'$ for all i. By definition, $v(\pi_i) = p^{-i}$. By construction we have $\mathit{inf}\, v(I') = 0$ and therefore there exists for all $i \in \mathbb{N}$ a $w_i \in I'$ such that $0 < v(w_i) < p^{-i}$. Hence, π_i is a multiple of w_i and $\pi_i \in I'$. Therefore, $J \subseteq I' \subseteq J$.

We now turn back to the general setup of the lemma.

Let, as usual, π_γ be a prime element of Δ_γ we also choose primes $p_\gamma \in R_\gamma$. We know that we may choose π_γ such that $\pi_\gamma^n = p_\gamma$.

We need one more observation.

Claim 5. Let $\gamma > \gamma_-$ and let e be the ramification index of R_γ over R_{γ_-}. Then $\pi_\gamma^e\Delta_\gamma = \pi_{\gamma_-}\Delta_\gamma$.

Proof of Claim 5. Recall that an extension of the valuation v_{γ_-} of R_{γ_-} to a valuation on R_γ is given by $v_{\gamma_-}(N_{[K_\gamma:K_{\gamma_-}]}(x)) =: v_\gamma(x)$. Recall moreover, that the valuation on Δ_{γ_-}, also denoted by v_{γ_-} is given by $v_{\gamma_-}(N_{[(K_{\gamma_-}\Delta_{\gamma_-}):K_{\gamma_-}]}(x))/n^2 =: v_{\gamma_-}(x)$ and that on Δ_γ is defined analously.

Obviously, for $x \in \Delta_{\gamma_-}$, we have $N_{[(K_{\gamma_-}\Delta_{\gamma_-}):K_{\gamma_-}]}(x) = N_{[(K_\gamma\Delta_\gamma):K_\gamma]}(x)$ since we may take the same basis over the centre. Henceforth,

$$v_\gamma(\pi_{\gamma_-}) = v_\gamma(N_{[(K_\gamma\Delta_\gamma):K_\gamma]}(\pi_{\gamma_-}))/n^2 = e\cdot v_{\gamma_-}(N_{[(K_{\gamma_-}\Delta_{\gamma_-}):K_{\gamma_-}]}(\pi_{\gamma_-}))/n^2 = v_\gamma(\pi_\gamma^e). \blacksquare$$

This completes the proof of the claim.

Let $X = \sum_{i\in\mathbb{N}} \Delta \cdot u_i$ be the non finitely generated ideal of Δ. We have a valuation v defined on the whole of $K\Delta$. Let $m := \mathit{inf}\,\{v(u_i) \mid i \in \mathbb{N}\}$ (observe that if there is an $x \in X$ with $m = v(x)$, then X is principal.) So, we take an element $u \in \Delta$ (observe that we are in a complete situation) with $v(u) = m$. Then, since $v(X \cdot u^{-1}) \geq 0$, all of the elements in $X \cdot u^{-1} =: X'$ are integral.

We claim that $X' = J$. Recall, $J = \sum_\alpha \pi_\alpha\Delta$. We have a local ring and therefore $X' \subseteq J$. We hence have to prove that $\pi_\alpha \in X'$ for all α. By construction we have $\mathit{inf}\, v(X') = 0$. Since X is not finitely generated, there exists a sequence in

$v(X) \setminus \{inf\, v(X)\}$ converging to $inf\, v(X)$. Hence, there exists a $w_\alpha \in X'$ for all α such that $0 < v(w_\alpha) < v(\pi_\alpha)$. Hence, π_α is a multiple of w_α and $\pi_\alpha \in X'$. Therefore, $J \subseteq X' \subseteq J$.

We describe the non trivial twosided ideals of Λ.

Multiplying from the left and the right by matrices $1_{i,j}$ which have entries only 0 except at one place (i, j) in the upper triangular submatrix where there is a 1, we see that the twosided ideals are of the following shape: I consists of the full matrix ring with entries in the i, j–th position in some twosided ideal $I_{i,j}$ of Δ satisfying some appropriate conditions determined in the sequel.

For all $r \in J$ we form the matrix $r_{i,j}$ consisting of 0 at every position except one in the lower triangular submatrix at the position i, j. multiplying by $r_{i,j}$ from the left and multiplying from the right, we see that

$$J \cdot I_{j,k} \subseteq I_{i,k} \text{ and } I_{k,i} \cdot J \subseteq I_{k,j}.$$

were $i < j$ and k arbitrary between 1 and n.

Multiplying by $1_{j,i}$ from the right and multiplying from the left, we get

$$I_{k,j} \subseteq I_{k,i} \text{ and } I_{i,k} \subseteq I_{j,k}.$$

Still, $i < j$ and k arbitrary. Hence, $I_{k,i} \cdot J \subseteq I_{k,j} \subseteq I_{k,i}$ and $J \cdot I_{j,k} \subseteq I_{i,k} \subseteq I_{j,k}$ for all k and $i < j$.

Firstly, if an ideal $I_{i,j}$ is finitely generated, then it is principal. In fact, all generators lie in Δ_α for large enough α, and hence, since the statement is true there, it is true in Δ.

Secondly, the above relations assure that surely if I is non zero then also all of the $I_{i,j}$ are non zero.

Third, for any ideal X different from 0 of Δ, the ideals $J \cdot X = X \cdot J$ are not finitely generated.

Fourth, $J^2 = J$.

So, take ideals $I_{i,j} = a_{i,j}J$ or $I_{i,j} = a_{i,j}\Delta_{i,j}$. The above relations ensure, that for all i, j we have $v(a_{i,j}) = v(a_{1,1}) = v(a)$ with $a := a_{1,1}$. If $I_{i,j} = a\Delta$, then $I_{k,l} = a\Delta$ for $k \geq i$ and $l \geq j$. All of these so formed ideals are however twosided ideals.

An application

In order to give an application to the derived category of Λ analogous to that in [K-Z] we recall the main result of [K-Z].

In [K-Z] there is given a complete and fully combinatorial description of the set of rings being derived equivalent to a given hereditary order over a complete discrete valuation domain. We sketch the result here. For the moment let Ω be a hereditary order over a complete discrete valuation domain S.

Let n be the number of simple Ω–modules. Write $n = n_1 + n_2$ as a sum of two non negative integers. We fix a circle with n distinguished vertices numbered consecutively from 1 to n, n_1 stars and n_2 arrows. Each star is attached to a vertex, each arrow goes from one vertex to another vertex. At each vertex there is at most one star. The arrows and stars are drawn in the circle obeying to the following rules:

(0) There is a star at 1 and if the number of the ending vertex of an arrow is smaller than the number of the starting vertex then the ending vertex is vertex 1.

(1) Two different arrows do not intersect in inner points.

(2) Each arrow lies on an open polygon path (that is a tree consisting of straight lines which are arrows) which contains exactly one star. There are no closed polygon paths.

(3) If an arrow leads from i to j, a star attached to some vertex in the circle segment between i and j can only be attached to i or to j.

The figures satisfying those rules are called *cascades of fans.* The rings attached to a cascade of fans is prescribed by the following procedure. For each vertex we write a matrix of size the number of arrows ending or starting at that vertex plus the number of stars (0 or 1) attached to that vertex. For each vertex we insert a $\overline{\Delta} := \Delta/J(\Delta)$ in the main diagonal, the arrows with starting vertex at the present one contribute a full lower triangular matrix, the arrows with ending vertex the present vertex contribute as upper triangular matrix. An occasionally present star is written in the first entry with a Δ and the positions that belong to the arrows starting at the present vertex we insert a $\overline{\Delta}$ at the first row, lower diagonal, at those ending at the vertex the dual procedure. We finally add a hereditary matrix in standard form of size "number of stars in the circle" and we take the subring of the direct sum of those matrix rings that consists of those elements with equal entries at those diagonal parts which belong to the same vertex or star. For more details and a more illustrative description we refer to [K-Z].

The rings derived equivalent to Λ are describable in almost the same manner as done in [K-Z]. We also get the description by cascades of fans as is done in [K-Z]. The proof there basically used only the fact that the projective indecomposable modules $P_1, P_2, ..., P_n$ have the property that $rad\ P_i \simeq P_{i-1}$ and $rad\ P_1 \simeq P_n$ as well as the fact that for the endomorphism rings Δ of the projective indecomposable modules P_i we have a structure as is proved for our Δ in Lemma 2. For the proof of [K-Z] it is also essential to have minimal mappings $P_i \longrightarrow P_j$ for all i, j. Since our $J(\Delta)$ for the semi–hereditary case is not finitely generated, we have a cascade of fans in a semicircle rather than a circle. In fact, there is no minimal mapping from

P_i to P_j if $j < i$. The rule (0) must be changed to (0_{semi}) and the rules (1_{semi}), (2_{semi}) and (3_{semi}) are as stated below.

(0_{semi}) There is at least one star and for all the arrows the index of the starting vertex is smaller than the index of the ending vertex.
For the other rules we have $(1_{semi}) = (1)$, $(2_{semi}) = (2)$ and $(3_{semi}) = (3)$.

The proof of [K-Z] carries then through word by word.

Corollary 1. If T is a tilting complex over Λ with endomorphism ring isomorphic to Λ, then T is isomorphic to a module, rather than a complex.

Proposition 2. ([Ric, Proposition 9.4]). If Λ and Γ are derived equivalent rings, and Λ is finitely generated as a module over a noetherian centre, then Γ is also finitely generated over a noetherian centre.

Example 2. The property that a ring is finitely generated as module over its centre is not preserved by derived equivalences if one does not have the centre being noetherian. Let

$$R = lim_{n\to\infty}\widehat{\mathbb{Z}}_p[\zeta_n]$$

where ζ_n is a primitive p^n th root of unity. The radical $J(R)$ of R is not finitely generated as R–module. Let Λ be the semi–hereditary order

$$\Lambda = \begin{pmatrix} R & \dots & \dots & R \\ J(R) & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ J(R) & \dots & J(R) & R \end{pmatrix}_{n\times n} .$$

The derived equivalence by a cascade of fans takes Λ to a ring which is finitely generated over its centre if and only if there is only one star in the cascade of fans. Observe that $R/J(R)$ is isomorphic to the prime field of p elements.

Corollary 2 The injective limit of a tilting complex need not be a tilting complex.

A cascade of fans with arrows ending at 1 for classical hereditary orders and the limit over these leads not to a tilting complex since for semihereditary orders we have a cascade of fans in the semicircle rather than a circle.

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