BATALIN-VILKOVISKY ALGEBRAS, TAMARKIN-TSYGAN CALCULUS AND ALGEBRAS WITH DUALITY; THE CASE OF FROBENIUS ALGEBRAS

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Abstract. This note reports on joint work with Thierry Lambre and Guodong Zhou. Let A be a Frobenius algebra with diagonalisable Nakayama automorphism. We exhibit a Tamarkin-Tsygan calculus on the Hochschild cohomology of A and Hochschild homology of A with values in the Nakayama twisted bimodule. Since this pair is an algebra with duality, as introduced by Lambre, these structures define a Batalin-Vilkovisky structure on the cohomology ring of A. We further give an easy and practical criterion when a Frobenius algebra has diagonalisable Nakayama automorphism.

1. Introduction

Hochschild cohomology $HH^*(A)$ and Hochschild homology $HH_*(A, M)$ with values in a bimodule M of an algebra has a very rich structure. First, the Hochschild cohomology is a graded commutative $\mathbb{N}$-graded algebra. Then, Gerstenhaber showed in [9] that the Hochschild cohomology algebra carries a graded Lie algebra structure, where the Lie bracket is graded in the sense $[\ ,\ ] : H^{n+1}(A) \times H^{m+1}(A) \to H^{n+m+1}(A)$. Moreover, these two structures are compatible in the sense that $[\alpha, -]$ is a graded derivation of the multiplicative structure. Structures of this kind are called Gerstenhaber algebras.

The Gerstenhaber bracket is somewhat mysterious and has been determined in only few cases. A nice description in terms of coderivations was given by Stasheff in [21]. If there is a differential Δ of degree -1 of a Gerstenhaber algebra such that the Gerstenhaber bracket is the obstruction of Δ to be a graded derivation of the Hochschild cohomology, then the Gerstenhaber algebra is called a Batalin-Vilkovisky algebra. This structure comes from theoretical physics, more precisely from quantum field theories as explained in e.g. [10].

In representation theory the Batalin-Vilkovisky structure was popularised by Ginzburg [11], where he proves that the Hochschild cohomology of a Calabi-Yau algebra A is a Batalin-Vilkovisky algebra. This result was generalised by Kowalzig and Krähmer to twisted Calabi-Yau algebras, i.e. there is n, such that the n-th syzygy of A as $A \otimes A^{op}$-module is ${}_1A_\alpha$ for some automorphism α of A, provided the twisting automorphism is diagonalisable. In a parallel development Tradler [23] showed that for symmetric algebras (i.e. k-algebras such that the k-linear dual of A is isomorphic to A as $A-A$-bimodule) the Hochschild cohomology also caries the structure of a Batalin-Vilkovisky algebra. In [17] Lambre, Zhou and Zimmermann show that the Hochschild cohomology ring of a Frobenius algebra is Batalin-Vilkovisky provided that the Nakayama automorphism is diagonalisable.

The detailed version of this paper will be submitted for publication elsewhere.

We shall report in this note about the various steps to the proof of this result. We will also give a short criterion which implies that the Nakayama automorphism of a Frobenius algebra is diagonalisable.

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2. Batalin-Vilkovisky algebras

We first give the precise definition of a Batalin-Vilkovisky algebra.

Definition 1.
- A *Gerstenhaber algebra* over a field k is the data $(\mathcal{H}^*, \cup, [\ ,\ ])$, where $\mathcal{H}^* = \oplus_{n\in\mathbb{Z}}\mathcal{H}^n$ is a graded k-vector space equipped with two bilinear maps

$$\cup\colon \mathcal{H}^n \times \mathcal{H}^m \to \mathcal{H}^{n+m}, \quad (\alpha,\, \beta) \mapsto \alpha \cup \beta$$

$$[\ ,\ ]\colon \mathcal{H}^{n+1} \times \mathcal{H}^{m+1} \to \mathcal{H}^{n+m+1}, \quad (\alpha,\, \beta) \mapsto [\alpha,\, \beta]$$

  called the cup product $\cup$, and the Lie bracket $[\ ,\ ]$ respectively such that
  - $(\mathcal{H}^*,\, \cup)$ is a graded commutative associative algebra with unit $1 \in \mathcal{H}^0$,
  - $(\mathcal{H}^*[-1],\, [\ ,\ ])$ is a graded Lie algebra,
  - for each homogeneous element $\alpha \in \mathcal{H}^*[-1]$ the map $[\alpha, -]$ is a graded derivation of the algebra $(\mathcal{H}^*, \cup)$.
- A Gerstenhaber algebra $(\mathcal{H}^*, \cup, [\ ,\ ])$ is a *Batalin–Vilkovisky algebra* (BV algebra for short) if there is an operator $\Delta\colon \mathcal{H}^* \to \mathcal{H}^{*-1}$ of degree -1 (called a generator of the Gerstenhaber bracket $[\ ,\ ]$) such that $\Delta \circ \Delta = 0$, $\Delta(1) = 0$, and $[\ ,\ ]$ is the obstruction for Δ to be a graded derivation of $(\mathcal{H}^*, \cup)$, i.e.

$$[\alpha, \beta] = (-1)^{|\alpha|+1}(\Delta(\alpha \cup \beta) - \Delta(\alpha) \cup \beta - (-1)^{|\alpha|}\alpha \cup \Delta(\beta)),$$

  for homogeneous elements $\alpha, \beta \in \mathcal{H}^*$.

Remark 2. Batalin-Vilkovisky algebras appeared in mathematical physics. As explained in [27] and [13] the Batalin-Vilkovisky algebra formalism is fully used in the closed string theory. As explained in [13] the Batalin-Vilkovisky structure gives an additional rigidity to the string theory, and a certain number of choices which have to be made in this theory respect this additional structure. More precisely, in string field theory one first chooses a conformal field theory [10, Definition 3.1]. This field theory defines a vector space, the state space, and a field is an element in this vector field. A string field theory action is written as a formal power series with values in the string field. Then, certain choices have to be made, linked to Feynman rules, and the physical observables are independent of these choices. [13] show that the relation between two string field actions arises from field transformations that are canonical with respect to the Lie bracket.

Some algebras have Hochschild cohomology rings which are Batalin-Vilkovisky algebras.

Theorem 3. *(Ginzburg* [11, Theorem 3.4.3]*) Let A be a Calabi-Yau algebra of dimension d. Then the Hochschild cohomology of A has the structure of a Batalin-Vilkovisky algebra.*

Ginzburg is actually much more precise. He constructs the map Δ explicitly, and obtains Δ from the dual of Connes' B-operator on the Hochschild homology complex, and conjugation by the isomorphism $HH^n(A) \simeq HH_{d-n}(A)$ which is deduced from the Calabi-Yau property. He also exhibits already there a connection to a Tamarkin-Tsygan calculus, in the same way as we will explain in Section 3.

In a parallel development Tradler considered more the case of finite dimensional algebras and proved that the Hochschild cohomology of symmetric algebras is a Batalin-Vilkovisky algebra.

Theorem 4. *(Tradler [23]) Let k be a field and let A be a finite dimensional symmetric k-algebra. Then $HH^*(A)$ is a Batalin-Vilkovisky algebra.*

The operator Δ is in this case the k-linear dual of Connes' B-operator, using that for symmetric algebras A we have $HH^n(A) \simeq Hom_k(HH_n(A), k)$ for all $n \in \mathbb{N}$. Note that the isomorphism uses the symmetrising form.

A next step was given by Kowalzig and Krähmer [15]. They generalise Ginzburg's result to a twisted version. For an automorphism α of an algebra A we denote by ${}_1A_\alpha$ the $A-A$-bimodule which is the regular A-module as left-module, but where the action of $a \in A$ from the right is given by multiplication with $\alpha(a)$. An algebra is twisted Calabi-Yau of dimension d if there is a class $\omega \in H_d(A, {}_1A_\alpha)$ such that $\omega_A \cap - : H^*(A, M) \to H_{d-*}(A, {}_1A_\alpha \otimes_A M)$ is an isomorphism (cf [12, Definition 3.6]).

Theorem 5. *(Kowalzig and Krähmer [15]) Let A be a twisted Calabi-Yau algebra of dimension d and twist α. If α acts as diagonalisable automorphism on the vector space A, then $HH^*(A)$ is a Batalin-Vilkovisky algebra.*

Kowalzig and Krähmer obtain in [15] a twisted version of Connes' map B, and use this twisted version to obtain Δ as its dual.

In joint work with Lambre and Zhou we shall be concerned with Frobenius algebras. These play the same role for symmetric algebras as twisted Calabi-Yau algebras do for Calabi-Yau algebras. Indeed, for Frobenius algebras we get an $A-A$-bimodule isomorphism $Hom_k(A, k) \simeq {}_1A_\nu$ for some automorphism ν of A, the Frobenius automorphism. Therefore, the k-linear dual of $HH^n(A)$ is not isomorphic to $HH_n(A)$, but rather to $HH_n(A, {}_1A_\nu)$, where ν is the Nakayama automorphism of A. For more ample details on Frobenius algebras see [26, Sections 1.10 and 4.5].

3. Twisting by automorphisms, the Tamarkin-Tsygan calculus

We shall not give directly the map Δ. Instead we shall prove that some parts of the Hochschild cohomology, together with the Hochschild homology, of a Frobenius algebra carries another important structure: It is a Tamarkin-Tsygan calculus, sometimes also called differential calculus.

Definition 6. A *Tamarkin-Tsygan calculus* is the data of $\mathbb{Z}$-graded vector spaces $\mathcal{H}^*$ and $\mathcal{H}_*$ together with graded bilinear inner laws $\cup$ and $[\ ,\ ]$ of $\mathcal{H}^*$ an a graded operation map $\cap$ of $(\mathcal{H}^*, \cup)$ on $\mathcal{H}_*$ such that

- $(\mathcal{H}^*, \cup, [\ ,\ ])$ is a Gerstenhaber algebra;

- $\mathcal{H}_*$ is a graded module over $(\mathcal{H}^*, \cup)$ via the map $\cap : \mathcal{H}_r \otimes \mathcal{H}^p \to \mathcal{H}_{r-p}, z \otimes \alpha \mapsto z \cap \alpha$ for $z \in \mathcal{H}_r$ and $\alpha \in \mathcal{H}^p$. That is, if we denote $\iota_\alpha(z) = (-1)^{rp} z \cap \alpha$, then $\iota_{\alpha \cup \beta} = \iota_\alpha \iota_\beta$;
- There is a map $B : \mathcal{H}_* \to \mathcal{H}_{*+1}$ such that $B^2 = 0$ and we have
$$L_\alpha \circ \iota_\beta - (-1)^{|\beta|} \iota_\beta \circ L_\alpha = \iota_{[\alpha,\beta]}$$
where we denote $L_\alpha = B \circ \iota_\alpha - (-1)^{|\alpha|} \iota_\alpha \circ B$.

It is not surprising to learn that [8] prove that Hochschild homology and cohomology give a Tamarkin-Tsygan calculus with the natural Gerstenhaber structure and a $\cap$ operation given by evaluation of the first terms of the Hochschild complex by some Hochschild cocycle. This coincides with the classical $\cap$-product well known in Hochschild theory. We note that the $\cap$ product can be defined as well on the action of $HH^*(A)$ on $HH_*(A, M)$ for any $A - A$-bimodule M, but it is not this Tamarkin-Tsygan structure that we use.

Remark 7. It would be nice to extend Stasheff's description [21] of the Gerstenhaber bracket by coderivations to the Tamarkin-Tsygan calculus on Hochschild (co-)homology.

Let α be an automorphism of the algebra A. We now develop the following very general construction. Recall the bar resolution $\mathbb{B}A$. Its degree n homogeneous component is $A^{\otimes n+2}$ and its differential b is given by $b_n(a_0 \otimes \cdots \otimes a_{n+1}) = \sum_{i=0}^{n} (-1)^i a_0 \otimes \cdots a_{i-1} \otimes a_i a_{i+1} \otimes a_{i+2} \otimes \cdots \otimes a_{n+1}$. It is well-known that this is a free $A \otimes A^{op}$-module resolution of A (cf e.g. [26]). The complex $Hom_{A \otimes A^{op}}(\mathbb{B}A, A)$ has homology $HH^*(A)$ and the homology of $\mathbb{B}A \otimes_{A \otimes A^{op}} {}_1A_\alpha$ is $HH_*(A, {}_1A_\alpha)$.

Observe that the degree n homogeneous component of $\mathbb{B}A \otimes_{A \otimes A^{op}} {}_1A_\alpha$ is isomorphic to $A^{\otimes n}$ and α acts diagonally on this space. Likewise, the degree n homogeneous component of $Hom_{A \otimes A^{op}}(\mathbb{B}A, A)$ is isomorphic to $Hom_k(A^{\otimes n}, k)$.

Since α is an algebra automorphism, $\alpha(1) = 1$ and so 1 is an eigenvalue of the action of α on A. It is easy to see that the eigenspace for the value 1 of the action of α on $Hom_{A \otimes A^{op}}(\mathbb{B}A, A)$, and on $\mathbb{B}A \otimes_{A \otimes A^{op}} {}_1A_\alpha$ respectively, are actually subcomplexes of $Hom_{A \otimes A^{op}}(\mathbb{B}A, A)$, and $\mathbb{B}A \otimes_{A \otimes A^{op}} {}_1A_\alpha$ respectively. Let $HH^*_{(1)}(A)$, respectively $HH^{(1)}_*(A, {}_1A_\alpha)$, be the corresponding homologies of these subcomplexes.

The structural maps $\cup$, $\cap$, $[\ ,\ ]$ do restrict to $HH^*_{(1)}(A)$ and to $HH^{(1)}_*(A, {}_1A_\alpha)$, which can be verified by an easy computations in a few lines.

Theorem 8. *(Lambre-Zhou-Zimmermann* [17]*) With the notation above, there is a degree* 1 *map* β_α *of* $HH^*_{(1)}(A)$ *such that* $(HH^*_{(1)}(A), \cup, [\ ,\ ], HH^{(1)}_{-*}(A, {}_1A_\alpha), \cap, \beta_\alpha)$ *is a Tamarkin-Tsygan calculus.*

We note that we need to use negative degrees for the homology part in order to get a formally correct calculus. The map β_α is much more tricky to obtain. It is an adaption of Kowalzig-Krähmer's map used in their proof.

4. Algebras with duality; the main result

The proofs we mentioned so far to prove that Hochschild cohomology is a Batalin-Vilkovisky algebra always used both, the Hochschild cohomology and the Hochschild homology, as well as some duality between them. Lambre formalised this in his concept of an algebra with duality.

Definition 9. (Lambre) An algebra with duality is given by $(\mathcal{H}^*, \cup, \mathcal{H}_*, \partial)$, where

- $(\mathcal{H}^*, \cup)$ is a graded commutative unitary algebra with unit $1 \in \mathcal{H}^0$,
- $\mathcal{H}_*$ is a graded vector space and c is an element of $\mathcal{H}_d$,
- ∂ is an isomorphism of vector spaces $\partial : \mathcal{H}_* \to \mathcal{H}^{d-*}$ satisfying $\partial(c) = 1$.

Observe that it is not really necessary to explicitly mention c. The third axiom implicitly defines it as image of 1 under ∂. Now, we come to the link between Tamarkin-Tsygan calculi and Batalin-Vilkovisky structures.

Proposition 10. *Let $(\mathcal{H}^*, \cup, \mathcal{H}_*, c, \partial)$ be an algebra with duality.*

(1) *We suppose that*
  (a) *$(\mathcal{H}^*, \cup, [\ ,\ ], \mathcal{H}_*, \cap, B)$ is a Tamarkin-Tsygan calculus,*
  (b) *the duality ∂ is a homomorphism of $\mathcal{H}^*$-right modules, i.e. we have the relation $\partial(z \cap \alpha) = \partial(z) \cup \alpha$.*

  Then the Gerstenhaber algebra $(\mathcal{H}^*, \cup, [\ ,\ ])$ is a BV-algebra with generator $\Delta = \partial \circ B \circ \partial^{-1}$.

(2) *We suppose that $(\mathcal{H}^*, \cup, [\ ,\ ], \Delta)$ is a BV-algebra with generator Δ. Then posing $B := \partial^{-1} \circ \Delta \circ \partial$ and $z \cap \alpha := \partial^{-1}(\partial(z) \cup \alpha)$, the data $(\mathcal{H}^*, \cup, [\ ,\ ], \mathcal{H}_*, \cap, B)$ is a Tamarkin-Tsygan calculus.*

If α acts as diagonalisable automorphism on A, then $Hom_{A\otimes A^{op}}(\mathbb{B}A, A)$ and $\mathbb{B}A \otimes_{A\otimes A^{op}} {}_1A_\alpha$ both decompose as a direct sum of eigenspace subcomplexes. Note however that we may get eigenvalues for the complexes which do not occur as eigenvalues for the action on A. This comes from the fact that if $A = \bigoplus_{\lambda\in\Lambda} A_\lambda$ is an eigenspace decomposition, then

$$A^{\otimes n} = \bigoplus_{(\lambda_1,\cdots,\lambda_n)\in\Lambda^n} A_{\lambda_1} \otimes \cdots \otimes A_{\lambda_n}.$$

The automorphism α acts on $A_{\lambda_1} \otimes \cdots \otimes A_{\lambda_n}$ with the eigenvalue $\lambda_1 \cdot \cdots \cdot \lambda_n$. Therefore if Λ is the set of eigenvalues of α, then the Hochschild complex decomposes as direct sum of subcomplexes which are eigenspaces for some $\lambda \in \langle\Lambda\rangle$, where $\langle\Lambda\rangle$ is the submonoid of the multiplicative group $k^\times$ of the base field generated by Λ. This decomposition is also the point where we use that α acts on A as diagonalisable automorphism.

Moreover, we get the most important formula on $\mathbb{B}A \otimes_{A\otimes A^{op}} {}_1A_\alpha$:

$$b \circ \beta_\alpha + \beta_\alpha \circ b = 1 - T$$

where T is the diagonal map of α on $A^{\otimes n}$ for each n, where b denotes the Hochschild differential and where β_α is defined in Theorem 8. Hence, only for the eigenspace of α for the eigenvalue 1 the corresponding subcomplex is not homotopic to 0. This shows

Proposition 11. *If α is diagonalisable, then $HH_*^{(1)}(A, {}_1A_\alpha) = HH_*(A, {}_1A_\alpha)$.*

We are almost done. Now suppose that A is a Frobenius algebra with Nakayama automorphism ν and consider the case $\alpha = \nu$. Then Theorem 8 and Proposition 11 provide a Tamarkin-Tsygan calculus on the Hochschild cohomology of a Frobenius algebra and the homology with values in the Nakayama twisted bimodule. Since

$$Hom_k(HH_n(A, {}_1A_\nu), k) \simeq HH^n(A)$$

we easily get an algebra with duality satisfying the hypotheses of the first part of Proposition 10. This shows

Theorem 12. *(Lambre, Zhou, Zimmermann* [17]*) Let k be a field and let A be a Frobenius k-algebra with diagonalisable Nakayama automorphism. Then $HH^*(A)$ is a Batalin-Vilkovisky algebra.*

Remark 13. Volkov obtained in [24] independently and at the same time a similar result by exhibiting the operator Δ by explicit computation on the Hochschild cocycles.

Remark 14. Let $\overline{k}$ be the algebraic closure of k and let $\overline{A} := \overline{k} \otimes_k A$. If A is a Frobenius k-algebra, then $\overline{A}$ is a Frobenius $\overline{k}$-algebra. We actually only need that the Nakayama automorphism of $\overline{A}$ acts as diagonalisable automorphism on $\overline{A}$.

5. Diagonalisable Nakayama automorphism

We are left with the question how we may verify when a Nakayama automorphism is diagonalisable. There is an easy case: If A is a Frobenius k-algebra and ν is of finite order n. Then the action of ν on A is a representation of the cyclic group of order n, and if n is invertible in k, then this group ring is semisimple. Hence, for large enough fields k with $nk = k$ we have that the action of ν is diagonalisable. This happens for example for finite dimensional Hopf algebras by a result of Radford [19] in combination with a result by Larson-Sweedler [18]. Also preprojective algebras of Dynkin type have this property. For quantum complete intersections it can be shown by a direct computation that there also we get a diagonalisable Nakayama automorphism.

What about more general basic Frobenius algebras? Consider basic algebras and let hence $A = kQ/I$ be a finite dimensional Frobenius algebra given by quiver with relations. We can choose a basis $\mathcal{B}$ of A consisting of paths which also contains a basis for the socle of each indecomposable projective A-module. Then by [14, Proposition 2.8], there is a natural choice of the defining bilinear form $\langle a, b\rangle = tr(ab)$ for $a, b \in A$ induced by the trace map

$$tr : A \to k, \quad p \in \mathcal{B} \mapsto \begin{cases} 1 & \text{if } p \in soc(A) \cap \mathcal{B} \\ 0 & \text{otherwise} \end{cases}$$

Then we show the following useful

Proposition 15. *(Lambre, Zhou, Zimmermann* [17]*) Assume that the basis $\mathcal{B}$ satisfies two further conditions:*

1. *for arbitrary two paths $p, q \in \mathcal{B}$, there exist another path $r \in \mathcal{B}$ and a constant $\lambda \in k$ such that $p \cdot q = \lambda r \in A$*
2. *for each path $p \in \mathcal{B}$, there exists a unique element $p^* \in \mathcal{B}$ such that $0 \neq p \cdot p^* \in soc(A)$*

If k is an algebraically closed field of characteristic 0 or of characteristic p with p strictly bigger than the number of arrows of Q. Then the two conditions (1) and (2) imply that the Nakayama automorphism of A is semisimple and the Hochschild cohomology of A is a BV algebra.

By a classification result of Asashiba [1] we get

Lemma 16. *Each self-injective algebra of finite representation type is Morita equivalent to an algebra kQ/I given by a quiver Q modulo admissible relations I verifying the conditions (1) and (2).*

An alternative proof can be given by the fact that each representation-finite algebra has a multiplicative basis (cf. [2]).

Lemma 17. *Basic special biserial algebras satisfy the hypotheses of Proposition 15.*

Finally, we were looking at algebras of polynomial growth. These were studied by Holm, Skowroński, Bocian, Białkowsky for a classification up to derived equivalences, and by Zhou and Zimmermann [25] up to stable equivalences, clearing also a few remaining cases in the derived equivalence classification. Also there we can show that almost all the cases satisfy the hypotheses of Proposition 15. The few remaining situations can be done by an elementary computation on the quiver, using the construction of Holm-Zimmermann [14] mentioned above.

We finish by mentioning that an easy computation shows that for a field k of characteristic 2 the self-injective Nakayama algebra with two simples and Loewy length 4 does not have a semisimple Nakayama automorphism action. The quiver of this Nakayama algebra has two arrows such that Lemma 16 shows that the hypothesis in Proposition 15 on the characteristic of the base field is indeed necessary.

Remark 18. I want to mention that the formula for the Frobenius bilinear form given by [14] was originally used to classify deformed preprojective algebras ([4], see also [5] for a rectification in case of type E) of type L_n up to derived equivalence. This was done using the so-called Külshammer structure, an additional structure on the degree 0 Hochschild homology of an algebra [3], linked to the p-power map. In joint work with Sorlin [20] we extended the classification to deformed preprojective algebras of type D_n. For the precise and somewhat technical definition of the deformation parameter see [4, Proposition 6.2] or [5, Exmple 10.6]. We computed the degree 0 Hochschild homology of deformed preprojective algebras of type D_n and showed that over an algebraically closed field the deformed preprojective algebra is never derived equivalent to the non deformed preprojective algebra. Indeed, the dimension of the degree 0 Hochschild homology of the deformed preprojective algebra, with deformation parameter k is at most $n + 2 + k$ for $k \leq n - 3$ whereas this dimension is $3n$ in the non-deformed case.

The preprojective algebras of generalised Dynkin type are also interesting with respect to the Tamarkin-Tsygan structure on the Hochschild (co-)homology. Ching-Hwa Eu computed this explicitly (cf [6, 7]).

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