

CATEGORICAL RESOLUTIONS OF BOUNDED DERIVED CATEGORIES

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Let \mathcal{C} be an abelian category. A contravariant functor F from \mathcal{C} to the category of abelian groups $\mathcal{A}b$ is called finitely presented, or coherent [A], if there exists an exact sequence

$$\mathrm{Hom}_{\mathcal{C}}(-, X) \longrightarrow \mathrm{Hom}_{\mathcal{C}}(-, Y) \longrightarrow F \longrightarrow 0$$

of functors. Let $\mathrm{mod}\text{-}\mathcal{C}$ denote the category of all coherent functors. The systematic study of $\mathrm{mod}\text{-}\mathcal{C}$ is initiated by Auslander [A]. He, not only showed that $\mathrm{mod}\text{-}\mathcal{C}$ is an abelian category of global dimension less than or equal to two but also provided a nice connection between $\mathrm{mod}\text{-}\mathcal{C}$ and \mathcal{C} . This connection, which is known as Auslander's formula [L, K], suggests that one way of studying \mathcal{C} is to study $\mathrm{mod}\text{-}\mathcal{C}$, that has nicer homological properties than \mathcal{C} , and then translate the results back to \mathcal{C} . In particular if we let \mathcal{C} to be $\mathrm{mod}\text{-}\Lambda$, where Λ is an artin algebra, Auslander's formula translates to the equivalence

$$\frac{\mathrm{mod}\text{-}(\mathrm{mod}\text{-}\Lambda)}{\{F \mid F(\Lambda) = 0\}} \simeq \mathrm{mod}\text{-}\Lambda$$

of abelian categories. As it is mentioned in [L], 'a considerable part of Auslander's work on the representation theory of finite dimensional, or more general artin, algebras can be connected to this formula'.

Recently, Krause [K] established a derived version of this equivalence. In my talk, some different (relative and derived) versions of this formula will be explained. Then I will give some applications of our results. Especially, by using a relative version of Auslander's formula, we show that bounded derived category of every artin algebra admits a categorical resolution. This, in particular, implies that bounded derived categories of artin algebras of finite global dimension determine bounded derived categories of all artin algebras.

REFERENCES

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