Third International Meeting on Integer-Valued Polynomials

29/11/2010 — 3/12/2010,

CIRM, Marseille, France

ABSTRACTS

Third International Meeting on Integer-Valued Polynomials and Problems in Commutative Algebra

COMBINATORIAL, ARITHMETICAL, ALGEBRAIC, TOPOLOGICAL AND DYNAMICAL ASPECTS

CIRM
International center of mathematics meetings,
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ON WEAKLY 2-ABSORBING IDEALS OF
COMMUTATIVE RINGS

Ayman BADAWI

Abstract: Let $R$ be a commutative ring with identity $1 \neq 0$. Various generalizations of prime ideals have been studied. For example, a proper ideal $I$ of $R$ is weakly prime if $a, b \in R$ with $0 \neq ab \in I$, then either $a \in I$ or $b \in I$. Also a proper ideal $I$ of $R$ is said to be 2-absorbing if whenever $a, b, c \in R$ and $abc \in I$, then either $ab \in I$ or $ac \in I$ or $bc \in I$. In this paper, we introduce the concept of a weakly 2-absorbing ideal. A proper ideal $I$ of $R$ is called a weakly 2-absorbing ideal of $R$ if whenever $a, b, c \in R$ and $0 \neq abc \in I$, then either $ab \in I$ or $ac \in I$ or $bc \in I$. For example, every proper ideal of a quasi-local ring $(R, M)$ with $M^3 = \{0\}$ is a weakly 2-absorbing ideal of $R$. We show that a weakly 2-absorbing ideal $I$ of $R$ with $I^2 \neq 0$ is a 2-absorbing ideal of $R$. We show that every proper ideal of a commutative ring $R$ is a weakly 2-absorbing ideal if and only if either $R$ is a quasi-local ring with maximal ideal $M$ such that $M^3 = \{0\}$ or $R$ is ring-isomorphic to $R_1 \times F$ where $R_1$ is a quasi-local ring with maximal ideal $M$ such that $M^2 = \{0\}$ and $F$ is a field or $R$ is ring-isomorphic to $F_1 \times F_2 \times F_3$ for some fields $F_1, F_2, F_3$.

Keywords: prime, weakly prime, 2-absorbing ideal, n-absorbing ideal

REFERENCES


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Abstract:

Let $D$ be a domain and $S = \langle n_1, \ldots, n_b \rangle = \{\sum_{i=1}^{b} a_i n_i | a_i \in \mathbb{N}\}$ be a numerical monoid. The numerical semigroup ring $D[S]$ of $D$ over $S$ is a subring of the standard polynomial ring $D[x]$ over $D$. Specifically, $D[S] = D[X^{n_1}, \ldots, X^{n_b}]$. This is a particular case of the general construction of a semigroup ring. Many ring theoretic and global factorization theoretic properties of semigroup rings have been studied, and a classic reference is [1]. We show that even on a local level, where we consider factorizations of individual elements, there is much that can be said about the factorization theory of $D[S]$.

Keywords: Semigroup rings, Numerical monoids, Length sets, Block monoids

References

BF-CONDITION FOR ONE-DIMENSIONAL ANALYTICALLY IRREDUCIBLE RINGS

Valentina BARUCCI

Abstract: A complete one-dimensional analytically irreducible ring $R$ is a subring of the ring of power series with coefficient in a field $k$, $V = k[[t]]$ (which is a DVR with a valuation $v$) but it also contains the DVR $W = k[[x]]$, where $xR$ is a minimal reduction of the maximal ideal $m$. Moreover each ideal of $R$ is a free module of rank $e$ over $W$, where $e$ is the multiplicity of the ring.

The $m$-adic filtration of $R$ is said to satisfy the BF condition if there exists a minimal reduction $xR$ of $m$ and a set of elements $\{f_0, \ldots, f_{e-1}\}$ of $R$ such that $\{v(f_0), \ldots, v(f_{e-1})\}$ is the Apery set of the numerical semigroup $v(R)$ and each power of $m$ is a free $W$-module generated by elements of the form $x^{h_0} f_j$. I will discuss in the talk some equivalent or weaker conditions and show that they are fulfilled for some classes of rings, in particular for semigroup rings. When the BF condition is satisfied, the semigroup filtration $v(m^i)$ gives remarkable information on the blowup $R' = \bigcup_{i \geq 0} (m^i : m^i)$ and on the associated graded ring $\bigoplus_{i \geq 0} m^i / m^{i+1}$.

Keywords: One-dimensional analytically irreducible ring, semigroup ring, associated graded ring
FACTORISATION DES POLYNÔMES EN UTILISANT LE RÉSULTANT

Abdessamad BELHADEF

Abstract: Dans cet exposé, on développe une méthode permettant de factoriser des polynômes à plusieurs variables en utilisant le résultant et en effectuant un calcul de pgcd. On déduit de cette méthode un algorithme de factorisation d’un polynôme à plusieurs variables.

Keywords: Polynômes, Irréductibilité.

References


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SUR UNE PROPRIÉTÉ DES POLYNÔMES DE NÖRLUND

Farid BENCHERIF

Abstract:
Soit \( (B_n(x)) \) la suite des polynômes de Nörlund définie par \( \sum_{n=0}^{\infty} B_n(x) \frac{z^n}{n!} = \left( \frac{z}{e^z-1} \right)^x \). Dans cette communication, nous prouvons que
\[
B_n(x) = \frac{(-1)^n}{d_n} x^{1+\text{mod}(n,2)} P_n(x)
\]
où \((d_n)_n\) est la suite d’entiers (répertoriée A001898 dans l’OEIS [4]), définie par
\[
d_n = M_n n!
\]
de \((M_n)_n\) la suite des nombres de Minkowski (répertoriée A053657 dans l’OEIS [4]), définie par
\[
M_n := \prod_{p \text{ premier}} p^{\lfloor \frac{n}{p-1} \rfloor + \lfloor \frac{n}{p(p-1)} \rfloor + \lfloor \frac{n}{p^2(p-1)} \rfloor + \cdots}
\]
et où \(P_n(x)\) est un polynôme primitif de \( \mathbb{Z}[x] \) de degré \( 2 \lfloor \frac{n}{2} \rfloor - 1 \) définie par
\[
P_n(x) = \sum_{k=0}^{2\lfloor \frac{n}{2} \rfloor - 1} A(n,k)x^k
\]
là famille d’entiers \((A(n,k))\) vérifiant pour \( n \) pair les relations :
\[
\frac{A(n,n-1-k)}{A(n+1,n-1-k)} = \begin{cases} 
  n + 1 - 2k & \text{si } k \in \{0,1,2\} \\
  1 & \text{si } k \in \{n-1,n-2\}
\end{cases} , \quad n \geq 2
\]
Cette étude réalisée grâce à un résultat de J-L Chabert [2] prolonge une étude sur les polynômes de Stirling exposée aux 26-ième Journées arithmétiques de Saint-Etienne [1], répondant à une question posée dans [3].

References

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ON GORENSTEIN GLOBAL DIMENSION

Driss BENNIS

Abstract:

The Gorenstein global dimension of rings is introduced in [3] as a refinement of the classical global dimension. In this talk, we present some results on the Gorenstein global dimension, especially those established in [1] and [2]. We conclude with a brief discussion of the scope and limits of our results.

Keywords: Gorenstein projective, injective, and flat dimensions; Gorenstein global dimension; Gorenstein rings; Pullback rings.

REFERENCES


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NEWTON AND SCHINZEL SEQUENCES IN QUADRATIC FIELDS

Paul-Jean CAHEN

Abstract: This is a joint work with David ADAM.
We consider a quadratic extension of a global field and give the maximal length of a Newton sequence, that is, a simultaneous ordering in Bhargava’s sense [2] or a Schinzel sequence [3], that is, that satisfies the condition of Brownin-Schinzel problem. In the case of a number field \( \mathbb{Q}(\sqrt{d}) \), we show that the maximal length of a Schinzel sequence is 1, except in seven particular cases, and explicitly compute the maximal length of a Schinzel sequence in these special cases. We show that Newton sequences are also finite, except for at most finitely many cases, all real, and such that \( d \equiv 1 \pmod{8} \). For \( d \not\equiv 1 \pmod{8} \), we show that the maximal length of a Newton sequence is 1, except in five particular cases, and again explicitly compute the maximal length in these special cases. In the case of a quadratic extension of a function field \( \mathbb{F}_q(T) \), we similarly show that, unless the ring of integers is isomorphic to some function field (in which case there are obviously infinite Newton and Schinzel sequences), the maximal length of a Schinzel sequence is finite and in fact, equal to \( q \). For imaginary extensions, Newton sequences are known to be finite [1] (unless the ring of integers is isomorphic to some function field). We show here that the same holds in the real case, but for finitely many extensions.

Keywords: Integer-Valued Polynomials, Newton and Schinzel sequences, quadratic number and function fields, simultaneous ordering.

REFERENCES
Abstract: Let $\mathbb{F}_q$ be a finite field with $q$ elements. Let $\mathbb{F}_q((T^{-1}))$ be the completion of $\mathbb{F}_q(T)$ for the infinite $1/T$-adic valuation $v_\infty$ and let $\Omega$ be the completion of an algebraic closure of $\mathbb{F}_q((T^{-1}))$ for the extension $v$ of $v_\infty$ to that algebraic closure. The field $\Omega$ is algebraically closed. The ring $\mathbb{F}_q[T]$ is the analog of the ring $\mathbb{Z}$, the field $\mathbb{F}_q(T)$ is the analog of the field $\mathbb{Q}$ of rational numbers, and the field $\mathbb{F}_q((T^{-1}))$ is the analog of the field of real numbers $\mathbb{R}$. Although the analogy does not work as well as before, the field $\Omega$ is viewed as the analogue of the field $\mathbb{C}$ of complex numbers. The degree map extends to $\Omega$ with $\deg(z) = -v(z)$ for all $z \in \Omega$. Let

$$f(X) = \sum_{n=0}^\infty c_n X^n$$

be an entire function on $\Omega$. For $r$ a real number, define the number

$$M(f, r) = \sup\{\deg(f(z)) : \deg(z) \leq r\}.$$

Let $S \subset \mathbb{F}_q[T]$. In this talk we give examples of results proved of the type: If $f$ satisfies the conditions

- $f(X) \in \mathbb{F}_q[T]$ for each $X \in S$,
- $\limsup(M(f, r)/q^r) < c(S)$, with $c(S)$ a constant depending on the set $S$,

then $f$ is a polynomial. All the proofs make use of a good choice of a basis of the $\mathbb{F}_q[T]$-module $\text{Int}(S, \mathbb{F}_q[T])$. **Keywords:** Integer-Valued Polynomials, Entire functions, Pólya fields.

**REFERENCES**

AN OVERVIEW OF SOME RECENT DEVELOPMENTS
ON INTEGER-VALUED POLYNOMIALS

Jean-Luc CHABERT

Abstract: The purpose of my talk is to give an overview of recent developments on integer-valued polynomials in different fields: combinatorics, arithmetic, algebra, topology and dynamics.

Shortly before the second meeting on integer-valued polynomials which held in Marseille in 2000, Manjul Bhargava [1] introduced the notion of $P$-ordering which is so useful for the study of integer-valued polynomials on subsets of valuation domains, and for the construction of polynomial normal bases of ultrametric spaces of continuous functions on a compact space. During this meeting, he suggested possible extensions of these notions in order to study some spaces of analytic functions.

However, it is only shortly before this third meeting that Bhargava [2] published a very interesting paper in the Journal of the American Mathematical Society where he gave ingenious extensions of the notion of $P$-ordering in order to construct polynomial normal bases of several spaces of regular ultrametric functions.

I will speak about these strong last results. I will also try to give an idea of several other recent results, especially those that will not be mentioned by the speakers who could not come. Moreover, I will take this opportunity to recall some questions raised a few years ago in Cortona [3] and to speak about the possible answers that have been given during these last years. Finally, if I have time enough, I will end my talk with a few open problems hoping that some of the participants could solve some of them by the end of the meeting.

Keywords: Integer-Valued Polynomials, $P$-orderings, ultrametric analysis.

References


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EXAMPLES OF NON-NOETHERIAN ALMOST
DEDEKIND DOMAINS

Gyu Whan CHANG

Abstract: Let \( \mathbb{Q} \) be the field of rational numbers, \( \{ X_\alpha \} \) be a nonempty set of indeterminates over \( \mathbb{Q} \), \( D_n = \mathbb{Q}[[ (X_\alpha)^{1/2n} ]] \) for all integers \( n \geq 0 \), \( D = \cup_{n=0}^{\infty} D_n \), \( Q_\alpha = \cup_{n=0}^{\infty} (1 - (X_\alpha)^{1/2n}) D_n \), and \( S = \mathbb{Q}[\{X_\alpha\}] \setminus \cup_\alpha (1 - X_\alpha) \mathbb{Q}[\{X_\alpha\}] \). In this talk, we show that \( D_S \) is a non-Noetherian almost Dedekind domain and \( \text{Max}(D_S) = \{(1 + (X_\alpha)^{1/2n}) D_S | n \geq 1 \text{ and } \alpha \} \cup \{(Q_\alpha)_S | \alpha \} \), where each \( (Q_\alpha)_S \) is non-invertible.

This appears in my recent paper with Kang and Toan [1].

Keywords: PID, almost Dedekind domain.

REFERENCES
FACTORING INTEGER-VALUED POLYNOMIALS: A SURVEY

Scott CHAPMAN

Abstract: This talk will summarize some results in the literature concerning the non-unique factorization properties of integer-valued polynomials. The results we will consider are contained in the papers [2], [1] and [4] (which are outlined in [3]). We will focus on the results of [4]. Let $D$ be a unique factorization domain and $S$ an infinite subset of $D$. If $f(X)$ is an element in the ring of integer-valued polynomials over $S$ with respect to $D$ (denoted Int$(S, D)$), then we characterize the irreducible elements of Int$(S, D)$ in terms of the fixed-divisor of $f(X)$. The characterization allows us to show that every nonzero rational number $n/m$ is the leading coefficient of infinitely many irreducible polynomials in the ring Int$(\mathbb{Z}) = \text{Int}(\mathbb{Z}, \mathbb{Z})$. Further use of the characterization leads to an analysis of the particular factorization properties of such integer-valued polynomial rings. In the case where $D = \mathbb{Z}$, we are able to show that every rational number greater than 1 serves as the elasticity of some polynomial in Int$(S, \mathbb{Z})$ (i.e., Int$(S, \mathbb{Z})$ is fully elastic).

Keywords: Integer-Valued Polynomials, non-unique factorization.

REFERENCES


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Abstract: In a 2005 paper [1] Victor Buchstaber and Andrei Lazarev introduced an object which is, in a certain sense, a dual to the ring of integer-valued polynomials over an integral domain. We explain how this dual may be considered as an algebra of operators, give a construction of a basis, and discuss some examples.

Keywords: Integer-Valued Polynomials, Group Algebras, Bialgebras.

REFERENCES

STAR OPERATIONS IN RING EXTENSIONS

Said El BAGHDADI ¹

Abstract: This is a Joint work with D.F. Anderson and M. Zafrullah

Given (semi)star operations $*_D$ and $*_R$ on integral domains $D \subseteq R$, we say that $*_D$ and $*_R$ are compatible if $(IR)^*_R = (I*DR)^*_R$ for every nonzero fractional ideal $I$ of $D$. In this talk we give some results on compatibility of star operations in ring extensions. We mainly focus on the classical star operations $v$, $t$ and $w$.

Keywords: Star operation, domain extensions, compatibility.

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BIRING AND PLETHORY STRUCTURES ON INTEGER-VALUED POLYNOMIAL RINGS

Jesse ELLIOTT

Abstract:

All rings and algebras herein are assumed commutative with identity. Let $A$ be a ring. An $A$-bialgebra is an $A$-algebra object in the opposite category of the category of $A$-algebras. Equivalently, an $A$-bialgebra is an $A$-algebra $B$ together with a lift of the functor $\text{Hom}_A(B, -)$ from $A$-algebras to sets to a functor from $A$-algebras to $A$-algebras. For example, the polynomial ring $A[X]$ has a natural structure as an $A$-bialgebra. We examine conditions under which the ring $\text{Int}(D)$ of integer-valued polynomials on an integral domain $D$ has the structure of a $D$-bialgebra that is compatible with the $D$-bialgebra structure on $K[X]$, where $K$ is the quotient field of $D$. In particular, we show that $\text{Int}(D)$ has such a structure if the natural $D$-algebra homomorphism $\text{Int}(D) \otimes_D \text{Int}(D) \rightarrow \text{Int}(D^2)$ is an isomorphism and $\text{Int}(D)$ is flat as a $D$-module. This holds in particular if $D$ is a Krull domain or more generally a TV PVMD. We also study properties of the functor $\text{Hom}_D(\text{Int}(D), -)$ and its left adjoint.

Keywords: Integer-valued polynomial, integral domain, bialgebra, biring, plethory.
PROJECTIVE STAR OPERATIONS ON POLYNOMIAL RINGS OVER A FIELD

Alice FABBRI 1

Abstract:

This talk is based on a joint work with Dr. Olivier A. Heubo. We consider the polynomial ring $S := K[X_0, \ldots, X_n]$ over a field $K$ and the rings $R_i := K[\frac{X_0}{X_i}, \ldots, \frac{X_n}{X_i}]$ for $0 \leq i \leq n$. We introduce the notion of a projective star operation on $S$ and relate it to the classical star operations on the $R_i$’s. We show that the projective Kronecker function ring $\text{PKr}(S, \star)$ of $S$ is the intersection of the Kronecker function rings $\text{Kr}(R_i, \star_i)$, $0 \leq i \leq n$, where the $\star_i$’s are pairwise compatible e.a.b. star operations on the $R_i$’s and $\star$ is a projective star operation on $S$ built from the $\star_i$’s. Moreover, in this setting, the projective Kronecker function rings turn out to be function rings in the sense of Halter-Koch.

Keywords: Star operations, Kronecker function rings, Projective models.

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ON THE DYNAMICS OF \( x \mapsto x^p + a \) IN A LOCAL FIELD

Youssef FARES

Abstract: This is a joint work with David ADAM
We discuss the dynamics of \( \varphi_a : x \mapsto x^p + a \) in a local field \( K \) where \( p \) denotes the characteristic of the residue field. Using a result on factorials preservation [1], we show that the minimal subsets of such a dynamical system \( (K, \varphi_a) \) are cycles. We will see that the lengths of these cycles depend on the trace of \( a \) and on the cardinality of the residue field. Finally, we give a complete classification of the topological conjugacy classes of these dynamical systems.

Keywords: Bhargava’s Factorials, local field, dynamical system.

REFERENCES

THE ULTRAFILTER TOPOLOGY
ON SPACES OF VALUATION DOMAINS AND
APPLICATIONS – PART II

Carmelo Antonio FINOCCHIARO

Abstract: The aim of this talk is to present some results obtained in a joint paper with Marco Fontana and K. Alan Loper, continuing the talk by Marco Fontana.

Let $K$ be a field and $A$ be a subring of $K$. I will denote by $\text{Zar}(K|A)$ the set of all the valuation domains having $K$ as the quotient field and containing $A$. I will continue to present the properties of a topology on $\text{Zar}(K|A)$, called the ultrafilter topology, that refines the Zariski topology on $\text{Zar}(K|A)$ and makes it a compact Hausdorff space.

I will present applications of this topology to characterize some classes of integrally closed domains. Moreover, I will show that the operation to complete (in the sense of Zariski) an e.a.b. semistar operation corresponds to the compactification of a certain subspace of $\text{Zar}(K|A)$. Finally, I will define the dual topology (in the sense of Hochster) on $\text{Zar}(K|A)$ and I will characterize, in terms of the ultrafilter topology and of the dual topology, when two complete semistar operations are identical.

Keywords: Valuation domain, ultrafilter topology, dual topology, e.a.b. semistar operation, completion.
Abstract:

The aim of this talk (Part I) is to provide an introduction to a joint work with Carmelo A. Finocchiaro and K. Alan Loper [4]. Some of the results contained in this work will be presented in the subsequent talk (Part II) by Carmelo.

Let $K$ be a field and let $A$ be a subring of $K$. After discussing the motivations of the work, I will present some results concerning various topologies on $\text{Zar}(K|A)$, the space of valuation domains which have $K$ as quotient field and which have $A$ as a subring, with a view to some applications of these results to the representations of integrally closed domains as intersections of valuation overrings.

In case $A$ is the prime subring of $K$, then $\text{Zar}(K|A)$, denoted in this case simply by $\text{Zar}(K)$, includes all valuation domains with $K$ as quotient field. A first topological approach to the space $\text{Zar}(K)$ is due to Zariski that proved the quasi-compactness of this space, endowed with what is now called Zariski topology (see [12] and [13]). Later, it was proven that if $K$ is the quotient field of $A$ then $\text{Zar}(K|A)$, endowed with Zariski’s topology, is a spectral space (in the sense of Hochster [7]; see [2], [3], and, for instance, the appendix of [9]).

Finally, I will briefly introduce the ultrafilter topology on $\text{Spec}(A)$, the prime spectrum of a ring $A$ (see [5] and [11]), and on the space $\text{Zar}(K)$. In these spaces, I will show the relations among the ultrafilter topology, the classical Zariski topology and the constructible topology [1] (or, patch topology [7]). For instance, I will show that $\text{Zar}(K)$ equipped with the ultrafilter topology (or with the Zariski topology) is a spectral space, using the notion of $K$-functions rings introduced by Halter-Koch [8] for extending the classical notion of Kronecker function ring (see also [10]).

Keywords: Valuation domain, spectral space, constructible topology, ultrafilter topology, Kronecker function ring

References

Abstract:

I will give a survey on integer-valued polynomials of non-commutative algebras, in one or several variables, focussing on the points:

(1) how to transfer to the algebra case the continuity and compactness arguments that have been used to determine the spectrum of traditional rings of integer-valued polynomials;

(2) what can we say about interpolation in the case that the constant functions are not among the polynomial functions;

(3) how to generalize integer-valued polynomials in several variables to the non-commutative case, using polynomials in non-commuting variables.

Keywords: Integer-Valued Polynomials, Pólya fields.
CLASSES OF MODULES RELATED TO SEMISTAR OPERATIONS

Gabriele FUSACCHIA

Abstract: In this talk we show how the introduction of a semistar operation over an integral domain allows to define three classes of modules: semistar null, cosemistar and semistar neutral modules. After investigating their structure, we describe the behavior of these classes through the concept of torsion theory and its connection with semistar operations. In particular, we discuss how this affects the study of injective modules, showing that every choice of a semistar operation induces a canonical direct decomposition of injective modules in three summands, each belonging to one of the three classes, up to essential extensions.

Keywords: Semistar operations, Torsion theories

REFERENCES


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STABILITY AND REGULARITY WITH RESPECT TO STAR OPERATIONS

Stefania GABELLI

Abstract: This talk is based on the recent joint papers [1] and [2], with Giampaolo Picozza.

Given a star operation $\ast$ on a domain $R$, let $F_\ast(R)$ be the semigroup of fractional $\ast$-ideals of $R$, with respect to $\ast$-multiplication. Then $R$ is called $\ast$-stable if each ideal $I \in F_\ast(R)$ is $\ast$-invertible in the fractional overring $E(I) := (I : I)$ of $R$ (where $\ast$ is the restriction of $\ast$ to the set of fractional ideals of $E(I)$) and $R$ is called Clifford $\ast$-regular if each $I \in F_\ast(R)$ is von Neumann regular, that is $I = (I^2J)^\ast$ for some fractional ideal $J$ of $R$.

We study and put in relation these two notions, extending some results recently obtained by Bazzoni and Kabbaj-Mimouni when $\ast$ is the identity or the $t$-operation.

Keywords: Stability, Clifford regularity, star operation

References

SEMIGROUP-THEORETICAL CHARACTERIZATIONS OF ARITHMETICAL INVARIANTS WITH APPLICATIONS TO NUMERICAL MONOIDS AND KRULL MONOIDS

Pedro A. GARCÍA-SÁNCHEZ ¹

Abstract: This is a joint work with V. Blanco and A. Geroldinger, [1].

Arithmetical invariants—such as sets of lengths, catenary and tame degrees—describe the non-uniqueness of factorizations in atomic monoids.

We study these arithmetical invariants by the monoid of relations and by presentations of the involved monoids. The abstract results will be applied to numerical monoids and to Krull monoids.

Keywords: presentations for semigroups, catenary degree, tame degree, sets of lengths, numerical monoid, Krull monoid.

REFERENCES


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The K-Zero-Divisor Hypergraph

Amor HAOUAOUI

Abstract:

The concept of a zero-divisor graph of a commutative ring was introduced by Beck [3]. However, he lets all elements of R be vertices of the graph and was mainly interested in colorings. In [1], Anderson and Livingston introduced and studied the zero-divisor graph whose vertices are the non-zero zero-divisors, and the authors studied the interplay between the ring-theoretic properties of a commutative ring and the graph-theory properties of its zero-divisor graph. In [4] Ch. Eslahchi and A. M. Rahimi extend the concept of a zero-divisor of a commutative ring R to that of a k-zero-divisor and investigate the interplay between the ring-theoretic properties of R and the graph-theoretic properties of its associated k-uniform hypergraph \( H_k(R) \) and they study some examples of k-zero-divisors. In this talk we change the definition of a k-zero-divisor. So for a ring R we associate a new k-uniform hypergraph. Also, we define and study some properties about the notion of k-zero divisor and we give some examples of k-uniform hypergraph \( H_k(R) \) of a commutative ring R.

Keywords: zero divisor, graph-theory.

References


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NONNIL-NOETHERIAN RING AND THE SFT PROPERTY

Sana HIZEM

Abstract: This is a joint work with Ali BENHISSI

A commutative ring $R$ is said to be nonnil-Noetherian if every ideal which is not contained in the nilradical of $R$ is finitely generated. We show that many of the properties of Noetherian rings are true for nonnil-Noetherian rings. Then we study the rings of formal power series over a nonnil-Noetherian ring. We prove that if $R$ is an SFT nonnil-Noetherian ring then $\dim R[[X_1, ..., X_n]] = \dim R + n$ and that the ring $R[[X_1, ..., X_n]]$ is also SFT. We prove that for a commutative ring $R$, $\text{Nil}(R)([X_1, ..., X_n]) = \text{Nil}(R[[X_1, ..., X_n]])$ if and only if $\text{Nil}(R)$ is an SFT ideal of $R$, and in that case $\text{Nil}(R[[X_1, ..., X_n]])$ is also an SFT ideal of $R[[X_1, ..., X_n]]$.

Keywords: Nonnil-Noetherian rings, Formal power series.

REFERENCES


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SUPER-ADDITIVE SEQUENCES AND ALGEBRAS OF INTEGER VALUED POLYNOMIALS

Keith JOHNSON

Abstract: Let $K$ be a field with discrete valuation $\nu$ and $D = \{ a \in K : \nu(a) \geq 0 \}$. Any algebra $D[x] \subseteq A \subseteq K[x]$ has associated to it its characteristic sequence of fractional ideals $\{I_n : n = 0, 1, 2, \ldots \}$ with $I_n$ consisting of 0 and the leading coefficients of elements of $A$ of degree no more than $n$ and its characteristic sequence of integers $\{a(n) : n = 0, 1, 2, \ldots \}$ with $a(n) = -\nu(I_n)$. We will discuss how combinatorial properties of this integer sequence reflect algebraic properties of $A$ with particular attention to the case $A = \text{Int}(S, D)$ of integer valued polynomials on a subset $S$ of $D$.

Keywords: Integer-Valued Polynomials, super-additive sequence

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Abstract: We assume that all rings are commutative and noetherian with identity throughout this talk.

Definition. Let $N^\bullet$ be an object of the derived category $D(R)$. We say that $N^\bullet$ is $J$-cofinite, or for short cofinite, if there exists $M^\bullet \in D_{ft}(R)$, such that $N^\bullet \simeq D_J(M^\bullet)$ in $D(R)$. Here $D_J(\cdot)$ is the $J$-dualizing functor.

In this talk, we shall prove the following theorems:

**Theorem 1.** Let $(A,\mathfrak{m})$ be a local ring, and $I$ an ideal of $A$. We denote by $M(A, I)_{cof}$ the set of $A$-modules $N$ satisfying the condition

$$(*) \quad \text{Supp}_A(N) \subseteq V(I) \quad \text{and} \quad \text{Ext}^j_A(A/I, N) \quad \text{is of finite type, for all } j.$$

If $I$ is an ideal of $A$ of dimension one, then the category $M(A, I)_{cof}$ forms an abelian subcategory of the category $M(A)$ of all $A$-modules.

**Theorem 2.** Let $(R, \mathfrak{n})$ be a regular local ring, and $J$ an ideal of $R$ of dimension one. Let $N^\bullet$ be in the derived category $D^+(R)$ and suppose that $R$ is complete with respect to the $J$-adic topology. Then $N^\bullet$ is $J$-cofinite if and only if $H^i(N^\bullet)$ is in $M(R, J)_{cof}$ for all $i$.

These results are concerned with the questions in [3, §2].

Delfino and Marley proved that $M(A, P)_{cof}$ is an abelian subcategory for a prime ideal $P$ of dimension one over a complete local ring $A$ (cf. [2, Theorem 2, p. 49]). Melkersson proposed the related results (cf. [6, Theorem 7.4, p. 664]).

Further we will announce the related topics about above theorems, if we had time (cf. [1], [4]).

**Keywords:** Abelian category, Derived category, Cofinite complex, Cofinite module

**References**


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SFT-INSTABILITY AND KRULL DIMENSION IN POWER SERIES RING OVER AN ALMOST PSEUDO-VALUATION DOMAIN

Mohamed KHALIFA

Abstract: An integral domain $R$ is said to be an almost pseudo-valuation domain (for short, APVD) if every prime ideal $P$ of $R$ is strongly primary (i.e., whenever $x, y \in q.f(R)$ and $xy \in P$, we have $x \in P$ or $y^n \in P$ for some integer $n \geq 1$). In this talk, we give a necessary and sufficient conditions on a finite-dimensional APVD $R$ so that the power series ring $R[[x_1, ..., x_n]]$ has finite Krull dimension. Also, we give a necessary and sufficient conditions on an APVD $R$ so that $R[[x_1, ..., x_n]]$ has the SFT-property. We give examples showing the instability of the SFT-property via power series ring over an APVD.

Keywords: Krull dimension, Power series ring, SFT-ring, almost pseudo-valuation domain (APVD).

REFERENCES


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Abstract: Let $R$ be a one dimensional analytically irreducible ring and let $I$ be an integral ideal of $R$. We study the relation between the irreducibility of the ideal $I$ in $R$ and the irreductibility of the corresponding semigroup ideal $v(I)$. It turns out that if $v(I)$ is irreducible, then $I$ is irreducible, but the converse does not hold in general. In this talk, we give some condition on $I$ and on $R$ for getting the converse and several examples will be given. The aim of my talk is to speak about the following interesting question that one may find in the following interesting paper [1]:

Keywords: Numerical semigroup, Semigroup ring, irreducible ideal.

REFERENCES

Abstract:

In this thesis, we focus on the set $\text{Int}(\mathcal{O}_K)$ of integer-valued polynomials over $\mathcal{O}_K$, the ring of integers of a number field $K$. According to G. Pólya, a basis $(f_n)_{n \in \mathbb{N}}$ of the $\mathcal{O}_K$-module $\text{Int}(\mathcal{O}_K)$ is said to be regular if for each $n \in \mathbb{N}$, $\deg(f_n) = n$. A field $K$ such that $\text{Int}(\mathcal{O}_K)$ has a regular basis is said to be a Pólya field and the Pólya group of number field $K$ is a subgroup of the class group of $K$ which can be considered as a measure of the obstruction for a field being a Pólya field.

We study the Pólya group of a compositum $L = K_1K_2$ of two galoisian extensions $K_1/\mathbb{Q}$ and $K_2/\mathbb{Q}$ and we link it to the behaviour of the ramification of primes in $K_1/\mathbb{Q}$ and $K_2/\mathbb{Q}$. We apply these results to number fields with small degree in order to enlarge the well known family of quadratic Pólya fields.

Furthermore, a field $K$ is a Pólya field if the products of all maximal ideals of $\mathcal{O}_K$ with the same norm are principal. Analogously to the classical embedding problem, we can set the following problem : is every number field contained in a Pólya field? We give a positive answer to this question : for each number field $K$, the Hilbert class field $H_K$ of $K$ is a Pólya field.

We know also that every ideal of $\mathcal{O}_K$ becomes principal in $\mathcal{O}_{H_K}$. This leads us to introduce the notion of Pólya extension : it is a field $L$ containing $K$ such that the Pólya group of $K$ becomes trivial by extension of ideals, it is also a field $L$ such that the $\mathcal{O}_L$-module generated by $\text{Int}(\mathcal{O}_K)$ has a regular basis. Consequently, $H_K$ is a Pólya extension of $K$ in the general case. Moreover, when $K$ is abelian, capitulation of ambiguous ideals of $K$ proves that the genus field of $K$ is a Pólya extension. This leads us to consider minimality and unicity questions for Pólya fields and Pólya extensions.

Keywords: Integer-Valued Polynomials, Pólya fields, Pólya group, Hilbert class field, Genus field, Pólya extensions.

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CHARACTERIZATIONS OF SOME INTEGRAL DOMAINS OF THE FORM $A + B[\Gamma^*]$  

Jung Wook LIM  

Abstract: 
Let $A \subseteq B$ be an extension of integral domains, $\Gamma$ be a nonzero torsion-free (additive) grading monoid with $\Gamma \cap -\Gamma = \{0\}$, $\Gamma^* = \Gamma - \{0\}$, $S$ be a (saturated) multiplicative subset of $A$, and let $R = A + B[\Gamma^*]$. In this talk, we find some equivalent conditions for $R$ to be a $Pv$MD, a GCD-domain, a GGCD-domain, a Bézout domain or a Prüfer domain when $A$ is a proper subset of $B = AS$ or $B$ contains the quotient field of $A$. 

This is a joint work with G.W. Chang and B.G. Kang.

Keywords: $A + B[\Gamma^*]$, Prüfer $v$-multiplication domain, $t$-splitting set, torsion-free grading monoid $\Gamma$ with $\Gamma \cap -\Gamma = \{0\}$.

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CONSTRUCTIVE LOCAL-GLOBAL PRINCIPLES IN COMMUTATIVE ALGEBRA

Henri LOMBARDI ¹

Abstract:
We discuss the use of some constructive local-global principles in order to transform abstract proofs (very frequent in commutative algebra) into constructive theorems.

E.g. in order to drop primes, maximal ideals, minimal primes present in many proofs.

Some of these principles are described in [1, Chapter 15]
Various examples will be given.

Keywords: Local-Global Principles.

REFERENCES

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ON FINITE SATURATED CHAINS OF OVERRINGS

Sebti MASSAOUĐ

Abstract:

If a domain $R$, with quotient field $K$, has a finite saturated chain of overrings from $R$ to $K$, then the integral closure of $R$ is a Prüfer domain. An integrally closed domain $R$ with quotient field $K$ has a finite saturated chain of overrings from $R$ to $K$ with length $m \geq 1$ iff $R$ is a Prüfer domain and $|\text{Spec}(R)| = m + 1$. In particular we prove that a domain $R$ has a finite saturated chain of overrings from $R$ to $K$ with length $\dim(R)$ if and only if $R$ is a valuation domain, and that an integrally closed domain $R$ has a finite saturated chain of overrings from $R$ to $K$ with length $\dim(R) + 1$ if and only if $R$ is a Prüfer domain with exactly tow maximal ideals such that at most one of them fails to contain every non-maximal prime. The relationship with maximal non-valuation subrings is also established.


REFERENCES


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RINGS OF INTEGER-VALUED POLYNOMIALS AND RATIONAL FUNCTIONS ON A SUBSET

Mi Hee PARK

Abstract: Let $D$ be a pseudo-valuation domain with associated valuation domain $V$ and let $E$ be a nonempty subset of the quotient field $K$. We investigate the algebraic structures of the rings $\text{Int}(E, D) \subseteq \text{Int}(E, V)$ of integer-valued polynomials and the rings $\text{Int}^R(E, D) \subseteq \text{Int}^R(E, V)$ of integer-valued rational functions.

Keywords: Precompact, integer-valued polynomial, integer-valued rational function, valuation domain, Prüfer domain, (globalized) pseudo-valuation domain.

REFERENCES


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PARAMETRIZATION OF INTEGRAL VALUES OF POLYNOMIALS

Giulio PERUGINELLI

Abstract: We give a complete classification of those integer-valued polynomials $f(X) \in \mathbb{Q}[X]$ whose image over the integers can be parametrized by a multivariate polynomial with integer coefficients, that is, the existence of $g \in \mathbb{Z}[X_1, \ldots, X_m]$ such that $f(\mathbb{Z}) = g(\mathbb{Z}^m)$. The necessary and sufficient condition for $f(X)$ is that $f \in \mathbb{Z}[B(X)]$ for some polynomial $B(X) = sX(sX - r)/2$, where $s$ and $r$ are coprime odd integers and $s$ is a prime power or it is equal to 1. In particular we obtain that 2 is the only prime factor dividing the common denominator $D$ of the coefficients of $f(X)$ and there exists a rational $\beta = r/s$ such that $f(X) = f(\beta - X)$. Moreover if $f(\mathbb{Z})$ is likewise parametrizable, then this can be done by a polynomial in one or two variables.

We will give also some ideas towards a generalization of this classification to number fields $K$, showing that the prime factors dividing $D$ are related to the order of the roots of unity contained in $K$ and to the $Y$-linear factors of the bivariate separated polynomial $f(Y) - f(X)$.

Keywords: Integer-Valued Polynomials, local field, dynamical system.
ARITHMETIC OF NON-PRINCIPAL ORDERS IN ALGEBRAIC NUMBER FIELDS

Andreas PHILIPP

Abstract: The maximal order $\mathcal{O}_K$ of an algebraic number field is a Dedekind domain, and its arithmetic is completely determined by its Picard group $\text{Pic}(\mathcal{O}_K)$. In particular, $\mathcal{O}_K$ is factorial if and only if its Picard group is trivial. In contrast, non-principal orders are not integrally closed, hence they are never factorial, and their arithmetic depends not only on their Picard group but also on the localizations at singular primes. A non-principal order $\mathcal{O}$ with $|\text{Pic}(\mathcal{O})| \geq 3$ inherits many arithmetical properties from the maximal order. In contrast, only little is known about the arithmetic of non-principal orders whose Picard group has at most two elements, even if all localizations are half-factorial. In this case, we formulate a new semigroup theoretic approach based on monoids of relations—see, for example, [2]. Using this machinery and common transfer principles as in [1, Chapter 3.2], we are able to give a quite explicit description of various arithmetical invariants such as the elasticity $\rho(\mathcal{O})$, the minimum distance $\min \Delta(\mathcal{O})$, the catenary degree $\mathfrak{c}(\mathcal{O})$, the monotone catenary degree $\mathfrak{c}_{\text{mon}}(\mathcal{O})$, and in some special situations even the tame degree $t(\mathcal{O})$. In particular, we prove that $\rho(\mathcal{O}) \in \left\{ 1, \frac{3}{2}, 2 \right\}$ and $\min \Delta(\mathcal{O}) \leq 1$.

Keywords: Non-unique factorizations, algebraic number fields, non-principal orders.

References

Abstract: New properties of divided domains are established by looking at multiplicatively closed subsets associated to ring morphisms. In particular, we show that divided domains have a lot of primary ideals and calculate the Krull associated prime ideals of an ideal. We show that divided domains are valuation subrings of their Prüfer hulls. We also show that the maximal spectrum of $(I : I)$ for an arbitrary ideal $I$ of a divided domain $R$ contracts to a subset of $\text{Ass}(I)$. We characterize some classes of integral domains which are also divided.

Keywords: Treed domains, divided domains.

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LENGTHS OF MAXIMAL CHAINS OF RINGS IN FCP EXTENSIONS

Martine PICAVET

Abstract: This is a joint work with Gabriel PICAVET

A ring extension $R \subseteq S$ of commutative rings is said to have FCP (for finite chain property) if each chain of $R$-subalgebras of $S$ is finite. The length $\ell[R, S]$ of the extension $R \subseteq S$ is the supremum of the lengths of chains of intermediary rings. The length of an FCP extension has already been gotten by several authors in special cases. The aim of my talk is to give the value of $\ell[R, S]$ for any FCP extension $R \subseteq S$. A key result is that $\ell[R, S] < \infty$ for any FCP extension $R \subset S$. In particular, if $\ell[R, S] = r$, for some integer $r$, there exists a maximal chain $R = R_0 \subset R_1 \subset \ldots \subset R_{r-1} \subset R_r = S$ of $R$-subalgebras of $S$ with length $r$. Another result shows that for an FCP extension $R \subseteq S$, we have $\ell[R, S] = \ell[R, \bar{R}] + \ell[\bar{R}, S]$, where $\bar{R}$ denotes the integral closure of $R$ in $S$. It is then natural to reduce the study of lengths to FCP integral extensions and to FCP integrally closed extensions. If $R \subset S$ is an integral FCP extension, we get that the greatest length of a maximal chain of $R$-subextensions of $S$ is gotten by a chain containing the t-closure of $R$ in $S$. So, we calculate the length of an FCP extension $R \subseteq S$ in the three following cases: (1) $R \subseteq S$ is infra-integral, (2) $R \subseteq S$ is t-closed and (3) $R \subseteq S$ is integrally closed. Then, for a general FCP extension $R \subseteq S$, the value of $\ell[R, S]$ is given.

Keywords: FCP extension, minimal extension, length of a chain of rings, seminormalization, t-closure, integral closure.

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THE INTEGRAL CLOSURE OF IDEALS AS A SEMISTAR OPERATION

Giampaolo PICOZZA

Abstract:
Star operations are an essential tool in multiplicative ideal theory. In particular, the $v$-operation (which associates to each ideal its bidual) and its associated operations denoted by $t$ and $w$ have been thoroughly studied and several classes of domains, such as Mori domains, Prüfer $v$-multiplication domains and Krull domains, have been defined or characterized using these operations.

Moreover, the equality of all combinations of two of these star operations has been studied, in order to characterize some relevant classes of domains.

The $b$-operation, which associates to an ideal its integral closure, has been one of the first examples of star operations. This operation provides also a clear justification for the introduction of semistar operations. Even if the integral closure of ideals has been deeply studied, in particular in the Noetherian context, not much has been done about its properties as a semistar operation.

In a joint work with Marco Fontana, we have studied the domains characterized by some properties of the $b$-operation. In this talk I will present some results on $b$-Noetherian domains (i.e., the domains in which the ascending chain condition on integrally closed ideals holds) and I will characterize the domains in which the equality between the $b$-operation and each of the other classical star operations holds.

Keywords: Star operations, Semistar operations, integral closure.
UNIQUE INTEGER-VALUED POLYNOMIAL IMAGE SETS

Vadim PONOMARENKO ¹

Abstract:
Given an integer-valued polynomial $f \in \text{Int}(\mathbb{Z})$, its image set $f(\mathbb{Z}) \subseteq \mathbb{Z}$ is the set of values it takes on at the integers. A natural question to ask is whether this set of values determines $f$ uniquely. Of course the answer is no, since $f(x + n)$ and $f(-x + n)$, for any $n \in \mathbb{Z}$, are also integer-valued polynomials with the same image set. Are these the only ones? The answer is usually yes but sometimes no; a complete answer will be given in the talk.

Keywords: Integer-Valued Polynomials

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TOWARDS A MORE PRECISE UNDERSTANDING OF
SETS OF LENGTHS

Wolfgang A. SCHMID

Abstract: For an element $a$ of an atomic domain or monoid, the set of lengths of $a$, denoted by $L(a)$, is the set of all integers $\ell$ such that there exists irreducibles $u_1, \ldots, u_\ell$ with $a = u_1 \ldots u_\ell$. By a result of A. Geroldinger [1] it is known that for Dedekind domains, and more generally Krull monoids, with finite class group the sets of lengths are ‘structured.’ More precisely, they are almost arithmetical multi-progressions (AAMPs) with global bounds on the parameters of these AAMPs. We present some results on a key-parameter of these AAMPs, namely the differences that can occur. Time permitting, applications will be mentioned.

Keywords: Non-unique factorizations, Dedekind domain, Krull monoid

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RINGS WHOSE IDEALS ARE COUNTABLY GENERATED

Phan Thanh TOAN

Abstract:
A ring $R$ is called an $\aleph_0$-ring if every ideal of $R$ is countably generated. We show that the power series ring $R[[X]]$ is an $\aleph_0$-ring if and only if it is a Noetherian ring.

Keywords: $\aleph_0$-ring, Noetherian ring, power series ring, countably generated ideal.

REFERENCES

Abstract:
In this article we describe the *prime spectrum*, the set of prime ideals, for certain two-dimensional rings of polynomials and power series. Our main result is the characterization of those partially ordered sets that arise as prime spectra of simple birational extensions of a power series ring in one indeterminate with coefficients in a countable Dedekind domain that has infinitely many maximal ideals.

**Keywords:** Prime ideals, birational extensions, power series rings.

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