The slice Burnside ring
and the section Burnside ring
of a finite group

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Overview

1. Biset functors

- Definition
- Applications

2. The Burnside functor

3. Slices
   - Morphisms of \( G \)-sets
   - The slice Burnside ring
   - The slice Burnside functor

4. Sections
   - Galois morphisms of \( G \)-sets
   - The section Burnside ring
   - The section Burnside functor

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Motivation

Example: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G, \text{Res}_G^H : R_C(G) \rightarrow R_C(H)$, and induction $\text{Ind}_G^H : R_C(H) \rightarrow R_C(G)$. When $\phi : G \rightarrow G'$, there is $\text{Iso}(\phi) : R_C(G) \rightarrow R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

When $N \triangleleft G$, there is an operation of inflation $\text{Inf}_G^{G/N} : R_C(G/N) \rightarrow R_C(G)$, and deflation $\text{Def}_G^{G/N} : R_C(G) \rightarrow R_C(G/N)$. These five operations endow $R_C$ with a structure of biset functor.
Motivation

Example:

\[ \begin{align*}
\text{Example:} \\
\text{Motivation:} \\
\text{Example:}
\end{align*} \]
Example: the correspondence $G \mapsto R_C(G)$
Motivation

- **Example**: the correspondence $G \mapsto R_C(G)$ has operations of restriction.

  When $\phi : G \cong \rightarrow G'$, there is $\text{Iso}(\phi) : R_C(G) \rightarrow R_C(G')$.

  These operations endow $R_C$ with a structure of Mackey functor.

  When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}_{G/N} : R_C(G/N) \rightarrow R_C(G)$ and deflation $\text{Def}_{G/N} : R_C(G) \rightarrow R_C(G/N)$.

  These five operations endow $R_C$ with a structure of biset functor.
Example: the correspondence $G \mapsto R^C(G)$ has operations of restriction $\forall H \leq G$.
Motivation

- **Example**: the correspondence $G \mapsto R_{\mathbb{C}}(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}^G_H : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(H)$.
Motivation

- **Example**: the correspondence $G \mapsto R^G(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R^G(G) \to R^G(H)$, and induction $\text{Ind}_H^G : R^G(H) \to R^G(G)$.
Example: the correspondence $G \mapsto \mathbb{R}_C(G)$ has operations of restriction $\forall H \leq G, \text{Res}_H^G : \mathbb{R}_C(G) \to \mathbb{R}_C(H)$, and induction $\text{Ind}_H^G : \mathbb{R}_C(H) \to \mathbb{R}_C(G)$.

When $\varphi : G \to G'$
**Example**: the correspondence $G \leftrightarrow R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}^G_H : R_C(G) \to R_C(H)$, and induction $\text{Ind}^G_H : R_C(H) \to R_C(G)$. When $\varphi : G \to G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. 

These operations endow $R_C$ with a structure of Mackey functor.

When $N \unlhd G$, there is an operation of inflation $\text{Inf}^G_G : R_C(G) \to R_C(G/N)$ and deflation $\text{Def}^G_G : R_C(G) \to R_C(G/N)$. These five operations endow $R_C$ with a structure of biset functor.
Motivation

- **Example**: the correspondence $G \leftrightarrow R_{\mathbb{C}}(G)$ has operations of restriction $\forall H \leq G, \text{Res}^G_H : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(H)$, and induction $\text{Ind}^G_H : R_{\mathbb{C}}(H) \to R_{\mathbb{C}}(G)$.

When $\varphi : G \to G'$, there is $\text{Iso}(\varphi) : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(G')$.

These operations fulfill various conditions of compatibility.
**Example**: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G, \ Res^G_H : R_C(G) \to R_C(H)$, and induction $\ Ind^G_H : R_C(H) \to R_C(G)$.

When $\varphi : G \xrightarrow{\cong} G'$, there is $Iso(\varphi) : R_C(G) \to R_C(G')$.

These operations fulfill various conditions of compatibility, e.g. transitivity conditions $\forall K \leq H \leq G, Res_K^H \circ Res_H^G = Res_K^G$. 
**Example**: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}^G_H : R_C(G) \to R_C(H)$, and induction $\text{Ind}^G_H : R_C(H) \to R_C(G)$.

When $\varphi : G \xrightarrow{\sim} G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$.

These operations fulfill various conditions of compatibility, e.g. transitivity conditions $\forall K \leq H \leq G$, $\text{Ind}^G_H \circ \text{Ind}^H_K = \text{Ind}^G_K$. 
Example: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G, \text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$. When $\varphi : G \rightarrow G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. These operations fulfill various conditions of compatibility, e.g. the Mackey formula

$$\forall H \leq G \geq K, \text{ Res}_H^G \circ \text{Ind}_K^G = \sum_{g \in [H \backslash G / K]} \text{Ind}_{H \cap g \cdot K}^H \circ \text{Iso}(c_g) \circ \text{Res}_{H \cap g \cdot K}^K.$$
Mackey functors

**Example:** the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$.

When $\varphi : G \xrightarrow{\cong} G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$.

These operations fulfill various conditions of compatibility, e.g. the Mackey formula

$$\forall H \leq G \geq K, \quad \text{Res}_H^G \circ \text{Ind}_K^G = \sum_{g \in [H \backslash G / K]} \text{Ind}_{H \cap gK}^H \circ \text{Iso}(c_g) \circ \text{Res}_{H \cap gK}^K.$$

These operations endow $R_C$ with a structure of Mackey functor.
Example: the correspondence $G \mapsto R_\mathbb{C}(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_\mathbb{C}(G) \to R_\mathbb{C}(H)$, and induction $\text{Ind}_H^G : R_\mathbb{C}(H) \to R_\mathbb{C}(G)$. When $\varphi : G \xrightarrow{\sim} G'$, there is $\text{Iso}(\varphi) : R_\mathbb{C}(G) \to R_\mathbb{C}(G')$. These operations endow $R_\mathbb{C}$ with a structure of Mackey functor.
Example: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $Res_H^G : R_C(G) \to R_C(H)$, and induction $Ind_H^G : R_C(H) \to R_C(G)$. When $\varphi : G \to G'$, there is $Iso(\varphi) : R_C(G) \to R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

When $N \triangleleft G$
Mackey functors

- **Example**: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$.

  When $\varphi : G \to G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$.

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- When $N \triangleleft G$, there is an operation of inflation $\text{Inf}_{G/N}^G : R_C(G/N) \to R_C(G)$.
Example: the correspondence $G \mapsto R_\mathbb{C}(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_\mathbb{C}(G) \to R_\mathbb{C}(H)$, and induction $\text{Ind}_H^G : R_\mathbb{C}(H) \to R_\mathbb{C}(G)$.

When $\varphi : G \xrightarrow{\cong} G'$, there is $\text{Iso}(\varphi) : R_\mathbb{C}(G) \to R_\mathbb{C}(G')$. These operations endow $R_\mathbb{C}$ with a structure of Mackey functor.

When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}_{G/N}^G : R_\mathbb{C}(G/N) \to R_\mathbb{C}(G)$, and deflation $\text{Def}_{G/N}^G : R_\mathbb{C}(G) \to R_\mathbb{C}(G/N)$. 

These five operations endow $R_\mathbb{C}$ with a structure of biset functor.
Example: the correspondence $G \mapsto R_{\mathbb{C}}(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(H)$, and induction $\text{Ind}_H^G : R_{\mathbb{C}}(H) \to R_{\mathbb{C}}(G)$.

When $\varphi : G \cong G'$, there is $\text{Iso}(\varphi) : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(G')$.

These operations endow $R_{\mathbb{C}}$ with a structure of Mackey functor.

When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}_{G/N}^G : R_{\mathbb{C}}(G/N) \to R_{\mathbb{C}}(G)$, and deflation $\text{Def}_{G/N}^G : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(G/N)$ (taking co-invariants by $N$).
**Example:** the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}^G_H : R_C(G) \to R_C(H)$, and induction $\text{Ind}^G_H : R_C(H) \to R_C(G)$.

When $\varphi : G \xrightarrow{\sim} G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$.

These operations endow $R_C$ with a structure of **Mackey functor**.

When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}^G_{G/N} : R_C(G/N) \to R_C(G)$, and deflation $\text{Def}^G_{G/N} : R_C(G) \to R_C(G/N)$.

These five types of operations fulfill a (long) list of **compatibility conditions**.
Mackey functors

**Example:** the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$. When $\varphi : G \xrightarrow{\sim} G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}_{G/N}^G : R_C(G/N) \to R_C(G)$, and deflation $\text{Def}_{G/N}^G : R_C(G) \to R_C(G/N)$. These five types of operations fulfill a (long) list of compatibility conditions, e.g. $\text{Def}_{G/N}^G \circ \text{Ind}_H^G = \text{Ind}_{HN/N}^{G/N} \circ \text{Iso}_{H/H \cap N}^{HN/N} \circ \text{Def}_{H/H \cap N}^H$.
Mackey functors

- **Example**: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$.

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  These five types of operations fulfill a (long) list of compatibility conditions.
Biset functors

- **Example**: the correspondence \( G \mapsto R_C(G) \) has operations of restriction \( \forall H \leq G, \text{Res}^G_H : R_C(G) \to R_C(H) \), and induction \( \text{Ind}^G_H : R_C(H) \to R_C(G) \).

When \( \varphi : G \xrightarrow{\sim} G' \), there is \( \text{Iso}(\varphi) : R_C(G) \to R_C(G') \).

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They can be unified using bisets.
Biset functors

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They can be unified using bisets: in each case, there are two finite groups $G$ and $H$. 

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Example: the correspondence $G \mapsto R_{\mathbb{C}}(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}^G_H : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(H)$, and induction $\text{Ind}^G_H : R_{\mathbb{C}}(H) \to R_{\mathbb{C}}(G)$. When $\varphi : G \leftrightarrow G'$, there is $\text{Iso}(\varphi) : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(G')$. These operations endow $R_{\mathbb{C}}$ with a structure of Mackey functor.

When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}^G_{G/N} : R_{\mathbb{C}}(G/N) \to R_{\mathbb{C}}(G)$, and deflation $\text{Def}^G_{G/N} : R_{\mathbb{C}}(G) \to R_{\mathbb{C}}(G/N)$. These five operations endow $R_{\mathbb{C}}$ with a structure of biset functor.

They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$. 
**Biset functors**

- **Example**: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \rightarrow R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \rightarrow R_C(G)$. When $\varphi : G \xrightarrow{\sim} G'$, there is $\text{Iso}(\varphi) : R_C(G) \rightarrow R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

- When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}_{G/N}^G : R_C(G/N) \rightarrow R_C(G)$, and deflation $\text{Def}_{G/N}^G : R_C(G) \rightarrow R_C(G/N)$. These five operations endow $R_C$ with a structure of biset functor.

- They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$, i.e. a set with a left action of $H$. When $\varphi : G \xrightarrow{\sim} H$, there is $\text{Iso}(\varphi) : R_C(G) \rightarrow R_C(H')$. These operations endow $R_C$ with a structure of biset functor.
Example: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$.

When $\varphi : G \cong G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

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They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$, i.e. a set with a left action of $H$, a right action of $G$. 

Biset functors
Example: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$. When $\varphi : G \xrightarrow{\sim} G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

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They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$, i.e. a set with a left action of $H$, a right action of $G$, which commute.
Biset functors

- **Example**: the correspondence \( G \mapsto R_\mathbb{C}(G) \) has operations of restriction \( \forall H \leq G, \text{Res}^G_H : R_\mathbb{C}(G) \rightarrow R_\mathbb{C}(H) \), and induction \( \text{Ind}^G_H : R_\mathbb{C}(H) \rightarrow R_\mathbb{C}(G) \).

  When \( \varphi : G \cong G' \), there is \( \text{Iso}(\varphi) : R_\mathbb{C}(G) \rightarrow R_\mathbb{C}(G') \).

  These operations endow \( R_\mathbb{C} \) with a structure of Mackey functor.

- When \( N \trianglelefteq G \), there is an operation of inflation \( \text{Inf}^G_{G/N} : R_\mathbb{C}(G/N) \rightarrow R_\mathbb{C}(G) \), and deflation \( \text{Def}^G_{G/N} : R_\mathbb{C}(G) \rightarrow R_\mathbb{C}(G/N) \).

  These five operations endow \( R_\mathbb{C} \) with a structure of biset functor.

- They can be unified using bisets: in each case, there are two finite groups \( G \) and \( H \), and a finite \((H, G)\)-biset \( U \), i.e. a set with a left action of \( H \), a right action of \( G \), which commute, i.e. such that \((hu)g = h(ug)\), \( \forall h \in H, u \in U, g \in G \).
Biset functors

- **Example**: the correspondence $G \leftrightarrow R_C(G)$ has operations of
  restriction $\forall H \leq G, \text{Res}_H^G : R_C(G) \to R_C(H)$, and
  induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$.

  When $\varphi : G \to G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$.

  These operations endow $R_C$ with a structure of Mackey functor.

- When $N \trianglelefteq G$, there is an operation of
  inflation $\text{Inf}_{G/N}^G : R_C(G/N) \to R_C(G)$, and
  deflation $\text{Def}_{G/N}^G : R_C(G) \to R_C(G/N)$.

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  groups $G$ and $H$, and a finite $(H, G)$-biset $U$. 
**Example**: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G, \text{Res}^G_H : R_C(G) \to R_C(H)$, and induction $\text{Ind}^G_H : R_C(H) \to R_C(G)$. When $\varphi : G \rightarrowtail G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

When $N \triangleleft G$, there is an operation of inflation $\text{Inf}^G_{G/N} : R_C(G/N) \to R_C(G)$, and deflation $\text{Def}^G_{G/N} : R_C(G) \to R_C(G/N)$. These five operations endow $R_C$ with a structure of biset functor.

They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$, such that the operation $R_C(U) : R_C(G) \to R_C(H)$.
Example: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}^G_H : R_C(G) \to R_C(H)$, and induction $\text{Ind}^G_H : R_C(H) \to R_C(G)$. When $\varphi : G \to G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

When $N \trianglelefteq G$, there is an operation of inflation $\text{Inf}_{G/N}^G : R_C(G/N) \to R_C(G)$, and deflation $\text{Def}_{G/N}^G : R_C(G) \to R_C(G/N)$. These five operations endow $R_C$ with a structure of biset functor.

They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$, such that the operation $R_C(U) : R_C(G) \to R_C(H)$ is induced by $W \mapsto \mathbb{C} U \otimes_{\mathbb{C} G} W$. 
**Example:** the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $Res^G_H : R_C(G) \to R_C(H)$, and induction $Ind^G_H : R_C(H) \to R_C(G)$.

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These operations endow $R_C$ with a structure of Mackey functor.

When $N \trianglelefteq G$, there is an operation of inflation $Inf^G_{G/N} : R_C(G/N) \to R_C(G)$, and deflation $Def^G_{G/N} : R_C(G) \to R_C(G/N)$.

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They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$, such that the operation $R_C(U) : R_C(G) \to R_C(H)$ is induced by $W \mapsto C U \otimes_C G W$.

Compatibility conditions
Example: the correspondence $G \mapsto R_C(G)$ has operations of restriction $\forall H \leq G$, $\text{Res}_H^G : R_C(G) \to R_C(H)$, and induction $\text{Ind}_H^G : R_C(H) \to R_C(G)$. When $\varphi : G \hookrightarrow G'$, there is $\text{Iso}(\varphi) : R_C(G) \to R_C(G')$. These operations endow $R_C$ with a structure of Mackey functor.

When $N \triangleleft G$, there is an operation of inflation $\text{Inf}_{G/N}^G : R_C(G/N) \to R_C(G)$, and deflation $\text{Def}_{G/N}^G : R_C(G) \to R_C(G/N)$. These five operations endow $R_C$ with a structure of biset functor.

They can be unified using bisets: in each case, there are two finite groups $G$ and $H$, and a finite $(H, G)$-biset $U$, such that the operation $R_C(U) : R_C(G) \to R_C(H)$ is induced by $W \mapsto C U \otimes_C G W$. Compatibility conditions $\Leftrightarrow R_C(V) \circ R_C(U) = R_C(V \times_H U)$. 
Biset functors

Definition

A biset functor $M$ consists of the following data:

1. For any finite group $G$, an abelian group $M(G)$,
2. For any finite $(H,G)$-biset $U$, a group homomorphism $M(U) : M(G) \to M(H)$,
3. Such that:
   - If $U \sim U'$ as $(H,G)$-bisets, then $M(U) = M(U')$.
   - $M(U_1 \sqcup U_2) = M(U_1) + M(U_2)$.
   - $M(V) \circ M(U) = M(V \times H U)$.
   - $M(\text{Id}_G) = \text{Id}_{M(G)}$.

More generally, one can consider biset functors with values in $k\text{-Mod}$, for a commutative ring $k$. Biset functors over $k$ form an abelian category $\mathcal{F}_k$. 

Serge Bouc (CNRS-LAMFA)
Biset functors

- When $G$, $H$, and $K$ are groups

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When $G$, $H$, and $K$ are groups, and $U$ is an $(H, G)$-biset, and $V$ is a $(K, H)$-biset, define $V \times H U = (V \times U) / H$. $V \times H U$ is a $(K, G)$-biset.

When $G$ is a group, let $\text{Id}_G$ denote the set $G$, viewed as a $(G, G)$-biset for left and right multiplication.

The Grothendieck group $B(H, G)$ of the category of finite $(H, G)$-bisets is called the Burnside group of $(H, G)$-bisets.

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Biset functors

When $G$, $H$, and $K$ are groups, when $U$ is an $(H, G)$-biset, and $V$ is a $(K, H)$-biset, define $V \times_H U = (V \times U)/H$. 

More generally, one can consider biset functors with values in $k\operatorname{-Mod}$, for a commutative ring $k$. Biset functors over $k$ form an abelian category $\mathcal{F}k$. 

Serge Bouc (CNRS-LAMFA)
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When $G$, $H$, and $K$ are groups, when $U$ is an $(H, G)$-biset, and $V$ is a $(K, H)$-biset, define $V \times_H U = (V \times U)/H$, where $(v, u)h = (vh, h^{-1}u)$ for $v \in V$, $u \in U$, $h \in H$. 
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Biset functors

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Biset functors
Biset functors

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Biset functors

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Biset functors

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More generally, one can consider biset functors with values in \( k\)-\( \text{Mod} \), for a commutative ring \( k \).

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Biset functors

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More generally, one can consider biset functors with values in $k\text{-}Mod$, for a commutative ring $k$. Biset functors over $k$ form an abelian category $\mathcal{F}_k$. 
Biset functors

Let $C_k$ be the following category:

The objects of $C_k$ are the finite groups.

If $G$ and $H$ are finite groups, then $\text{Hom}_{C_k}(G, H) = kB(H, G)$.

Composition in $C_k$ is the bilinear extension of $(V, U) \mapsto V \times H U$.

The identity morphism of the group $G$ is $\text{Id}_G \in kB(G, G)$.

A biset functor is a $k$-linear functor $C_k \to k$-Mod.

Examples:

1. The functors of complex representations $\mathbb{R} C$.
2. The Burnside functor $B$: $G \mapsto B(G) = B(G, 1)$.
   It is the Yoneda functor $\text{Hom}_{C_k}(1, -)$.
   In particular $B$ is projective in $F \mathbb{Z}$.
Biset functors

Equivalent definition

Let $C_k$ be the following category:

- The object of $C_k$ are the finite groups.
- If $G$ and $H$ are finite groups, then $\text{Hom}_{C_k}(G,H) = kB(H,G)$.
- Composition in $C_k$ is the bilinear extension of $(V,U) \mapsto V \times H U$.
- The identity morphism of the group $G$ is $\text{Id}_G \in kB(G,G)$.

A biset functor is a $k$-linear functor $C_k \to k$-Mod.

Examples:

1. The functors of complex representations $\mathbb{C}$.
2. The Burnside functor $B: G \mapsto B(G) = B(G,1)$.
   - It is the Yoneda functor $\text{Hom}_{C_k}(1,-)$.
   - In particular, $B$ is projective in $\mathbb{F}Z$.
Biset functors

Equivalent definition

Let $C_k$ be the following category:

- The object of $C_k$ are the finite groups.

Composition in $C_k$ is the bilinear extension of $(V, U) \mapsto V \times H U$.

The identity morphism of the group $G$ is $[\text{Id}_G] \in k B(G, G)$.

A biset functor is a $k$-linear functor $C_k \to k$-Mod.

Examples:

1. The functors of complex representations $\mathcal{R}$.
2. The Burnside functor $\mathcal{B}$:
   $G \mapsto B(G) = B(G, 1)$.
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   In particular $\mathcal{B}$ is projective in $F Z$.
Biset functors

Equivalent definition

Let $C_k$ be the following category:

- The object of $C_k$ are the finite groups.
- If $G$ and $H$ are finite groups, then $\text{Hom}_{C_k}(G, H) = k \otimes \mathbb{Z} B(H, G)$.

Examples:
1. The functors of complex representations $\text{RepC}$.
2. The Burnside functor $\text{B}$: $G \mapsto B(G) = B(G, 1)$. It is the Yoneda functor $\text{Hom}_{C_k}(1, -)$.

In particular $\text{B}$ is projective in $\text{F}_\mathbb{Z}$. 

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Biset functors

Equivalent definition

Let $C_k$ be the following category:

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Biset functors

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- Composition in $C_k$ is the bilinear extension of $(V, U) \mapsto V \times_H U$.

A biset functor is a $k$-linear functor $C_k \to k\text{-Mod}$.

Examples:

1. The functors of complex representations $R^C$.
2. The Burnside functor $B: G \mapsto B(G) = kB(G, 1)$. It is the Yoneda functor $\text{Hom}_{C_k}(1, -)$.

In particular $B$ is projective in $FZ$. 

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Biset functors

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A biset functor (with values in $k\text{-Mod}$) is a $k$-linear functor $C_k \to k\text{-Mod}$.
Equivalent definition

Let \( C_k \) be the following category :

- The object of \( C_k \) are the finite groups.
- If \( G \) and \( H \) are finite groups, then \( \text{Hom}_{C_k}(G, H) = kB(H, G) \).
- Composition in \( C_k \) is the bilinear extension of \( (V, U) \mapsto V \times_H U \).
- The identity morphism of the group \( G \) is \([\text{Id}_G] \in kB(G, G)\).

A **biset functor** is a \( k \)-linear functor \( C_k \rightarrow k\text{-Mod} \).

Examples :

- \((k = \mathbb{Z}, C = C\mathbb{Z})\)
- The functors of complex representations \( \mathbb{C} \).
- The Burnside functor \( B : G \mapsto kB(G, 1) \).
  It is the Yoneda functor \( \text{Hom}_{C_k}(1, -) \).
  In particular \( B \) is projective in \( \mathbb{F}_\mathbb{Z} \).
**Biset functors**

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Let $C_k$ be the following category:

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A **biset functor** is a $k$-linear functor $C_k \rightarrow k\text{-}\text{Mod}$.

**Examples**: $(k = \mathbb{Z}, C = C_\mathbb{Z})$
Biset functors

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Let $C_k$ be the following category:
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Examples: $(k = \mathbb{Z}, C = C\mathbb{Z})$
- The functors of complex representations $R_{\mathbb{C}}$. 
Let $C_k$ be the following category:

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**Examples:** $(k = \mathbb{Z}, C = C_\mathbb{Z})$

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Biset functors

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Examples:

1. The functors of complex representations $R_{\mathbb{C}}$.
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Examples: $(k = \mathbb{Z}, C = C_\mathbb{Z})$

1. The functors of complex representations $R_C$.
2. The Burnside functor $B : G \mapsto B(G) = B(G, 1)$. It is the Yoneda functor $\text{Hom}_C(1, -)$. In particular $B$ is projective in $F_\mathbb{Z}$. 
Some applications

Let $P$ denote a finite $p$-group. Rational representations of $P$. Genetic subgroups, genetic bases. Description of the kernel of the linearization morphism $B(P) \to RQ(P)$. Structure of the group of units $B(P) \times$. Structure of the Dade group $D(P)$. 

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Let $P$ denote a finite $p$-group.
Some applications

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Simple functors

Parametrization of simple functors

The correspondence $S \mapsto (H, V)$, where $H$ is a minimal group such that $S(H) \neq 0$, and $V = S(H)$, induces a bijection.

Simple biset functors with values in $k$-Mod up to isomorphism $\leftrightarrow$ Pairs $(H, V)$ such that $H$ is finite, $V$ is simple, and $kOut(H)$-module up to isomorphism.

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Simple biset functors
with values in $k$-$\textbf{Mod}$
up to isomorphism

Pairs $(H, V)$
$\begin{cases} 
H \text{ finite group} \\
V \text{ simple } k\text{Out}(H)\text{-module} 
\end{cases}$
up to isomorphism
Simple functors

Parametrization of simple functors

The correspondence $S \mapsto (H, V)$, where $H$ is a minimal group such that $S(H) \neq 0$, and $V = S(H)$, induces a bijection

Simple biset functors with values in $k$-$\text{Mod}$ up to isomorphism

Pairs $(H, V)$

$\begin{cases} 
H \text{ finite group} \\
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$(H, V)$
Simple functors

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The correspondence $S \mapsto (H, V)$, where $H$ is a minimal group such that $S(H) \neq 0$, and $V = S(H)$, induces a bijection

Simple biset functors with values in $k\text{-Mod}$ up to isomorphism

Pairs $(H, V)$

\begin{align*}
S_{H,V} &\leftrightarrow (H, V) \\
\{ &\begin{array}{l}
H \text{ finite group} \\
V \text{ simple } k\text{Out}(H)-\text{module}
\end{array} \\
&\text{up to isomorphism}
\end{align*}
Simple functors

Parametrization of simple functors

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Simple biset functors with values in $k$-$\text{Mod}$ up to isomorphism

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up to isomorphism
\end{align*}
Simple functors

Examples:

1. $S_1$, $k \sim = kR_Q$, and $kB$ is its projective cover.

2. $S_H$, $V$ subquotient of $kB \iff V = k$, and $H$ is a $B$-group.

This gives a way to compute $\dim_k S_H$, $k(G)$, when $H$ is a $B$-group, as the number of certain conjugacy classes of subgroups of $G$ (e.g. cyclic ones if $H = 1$).

4. In general $\dim_k S_H$, $V(G)$ is equal to the rank of a complicated matrix built from sections $(T, S)$ of $G$ (i.e. $S \triangleright T \leq G$) such that $T/S \sim = H$. The computation of this rank is usually very hard.
Simple functors

Examples:

1. $S$, $k \sim \mathbb{Q}$, and $kB$ is its projective cover.

2. $S\mathcal{H}$, $V$ subquotient of $kB$ $\iff V = k$, and $\mathcal{H}$ is a $B$-group.

3. This gives a way to compute $\dim_k S\mathcal{H}$, $k(G)$, when $\mathcal{H}$ is a $B$-group (e.g. cyclic ones if $\mathcal{H} = 1$).

4. In general $\dim_k S\mathcal{H}, V(G)$ is equal to the rank of a complicated matrix built from sections $(T, S) \leq G$ such that $T/S \sim H$. The computation of this rank is usually very hard.
Simple functors

Examples: \((k \text{ is a field, } \text{char}(k) = 0)\)
Examples: \((k \text{ is a field, } \text{char}(k) = 0)\)

1. \(S_{1,k}\)
Simple functors

Examples: (\(k\) is a field, \(\text{char}(k) = 0\))

1. \(S_{1,k} \cong kR_Q\)
Examples: $(k$ is a field, $\text{char}(k) = 0)$

1. $S_{1,k} \cong kR_{\mathbb{Q}}$, and $kB$ is its projective cover.
Simple functors

Examples: (\(k\) is a field, \(\text{char}(k) = 0\))

1. \(S_{1,k} \cong kR_\mathbb{Q}\), and \(kB\) is its projective cover.
2. \(S_{H,V}\) subquotient of \(kB\)
Examples: (k is a field, char(k) = 0)

1. $S_{1,k} \cong kR_Q$, and $kB$ is its projective cover.
2. $S_{H,V}$ subquotient of $kB \iff V = k$
Examples: \((k \text{ is a field, } \text{char}(k) = 0)\)

1. \(S_{1,k} \cong kR_{\mathbb{Q}}, \text{ and } kB \text{ is its projective cover.}\)
2. \(S_{H,V} \text{ subquotient of } kB \iff V = k, \text{ and } H \text{ is a } B\text{-group.}\)
Simple functors

Examples: \(k\) is a field, \(\text{char}(k) = 0\)
1. \(S_{1,k} \cong kR_{\mathbb{Q}},\) and \(kB\) is its projective cover.
2. \(S_{H,V}\) subquotient of \(kB \iff V = k,\) and \(H\) is a \(B\)-group.
3. This gives a way to compute \(\dim_k S_{H,k}(G),\) when \(H\) is a \(B\)-group.
Simple functors

Examples: (\(k\) is a field, \(\text{char}(k) = 0\))

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3. This gives a way to compute \(\dim_k S_{H,k}(G)\), when \(H\) is a \(B\)-group, as the number of certain conjugacy classes of subgroups of \(G\).
Examples: ($k$ is a field, $\text{char}(k) = 0$)

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Simple functors

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3. This gives a way to compute \( \dim_k S_{H,k}(G) \), when \( H \) is a \( B \)-group, as the number of certain conjugacy classes of subgroups of \( G \) (e.g. cyclic ones if \( H = 1 \)).

4. In general \( \dim_k S_{H,V}(G) \) is equal to the rank of a complicated matrix...
Examples: \((k\text{ is a field, } \text{char}(k) = 0)\)

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Examples: $(k$ is a field, $\text{char}(k) = 0)$

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Examples: \((k\text{ is a field, } \text{char}(k) = 0)\)

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Examples: \((k\) is a field, \(char(k) = 0\))

1. \(S_{1,k} \cong kR_{Q}\), and \(kB\) is its projective cover.
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The Burnside group

The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

$$B(G) = \mathbb{Z}\{[G\text{-sets}]\}/<[X \sqcup Y] - [X] - [Y]>.$$  

The group $B(G)$ is a free abelian group on $\{[G/H] | H \in \mathbb{S}_G\}$.

The cartesian product of $G$-sets induces a ring structure on $B(G)$.

There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in \mathbb{S}_G} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).

$Q B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

The prime spectrum of $B(G)$ can be explicitly described (Dress).

$\text{Spec } B(G)$ connected $\iff G$ is solvable.

The correspondence $G \mapsto B(G)$ is a Green biset functor.
The Burnside group $B(G)$ of a finite group $G$
The Burnside group is the Grothendieck group of finite $G$-sets, i.e. $B(G) = \mathbb{Z}\{\text{finite } G\text{-sets}\}/<\text{X \sqcup Y} - \text{X} - \text{Y}>$. The group $B(G)$ is a free abelian group on $\{G/H | H \in \text{s}_G\}$. The cartesian product of $G$-sets induces a ring structure on $B(G)$. There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in \text{s}_G} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969). $Q B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983). The prime spectrum of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\iff G$ is solvable. The correspondence $G \mapsto B(G)$ is a Green biset functor.
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The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

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The Burnside group

1. The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
$$B(G) = \mathbb{Z}\{[G\text{-sets}]\} / < [X \sqcup Y] - [X] - [Y] >.$$

2. The group $B(G)$ is a free abelian group on $\{[G/H] \mid H \in [s_G]\}$ (i.e. $H \leq G$, mod. $G$).
The Burnside group

1. The **Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
   
   $B(G) = \mathbb{Z}\{[G\text{-sets}]\} / \langle [X \sqcup Y] - [X] - [Y] \rangle$.

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The Burnside ring

1. **The Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
   
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3. The cartesian product of $G$-sets induces a **ring structure** on $B(G)$.

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Serge Bouc (CNRS-LAMFA)  
Slice - Section  
Pohang, March 28, 2011
The Burnside ring

1. The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
   \[ B(G) = \mathbb{Z}\{[G\text{-sets}]\}/< [X \sqcup Y] - [X] - [Y] >. \]

2. The group $B(G)$ is a free abelian group on $\{[G/H] \mid H \in [s_G]\}$.

3. The cartesian product of $G$-sets induces a ring structure on $B(G)$.

4. There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in [s_G]} \mathbb{Z}$.
The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

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The cartesian product of $G$-sets induces a ring structure on $B(G)$.

There is a ghost ring homomorphism $\Phi : B(G) \rightarrow \prod_{H \in [s_G]} \mathbb{Z}$, induced by $\phi_H : X \mapsto |X^H|$.
The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

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The cartesian product of $G$-sets induces a ring structure on $B(G)$.

There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$).
The Burnside ring

1. The **Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
   
   $B(G) = \mathbb{Z}\{[G\text{-sets}]\}/\langle [X \sqcup Y] - [X] - [Y] \rangle$.

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4. There is a **ghost ring homomorphism** $\Phi : B(G) \rightarrow \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).

5. $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

6. The prime spectrum of $B(G)$ can be explicitly described (Dress).

7. The correspondence $G \mapsto B(G)$ is a **Green biset functor**.

8. Serge Bouc (CNRS-LAMFA)
The Burnside ring

1. The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
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The Burnside ring

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2. The group $B(G)$ is a **free abelian group** on $\{[G/H] \mid H \in [s_G]\}$.

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The Burnside ring

1. The **Burnside group** \( B(G) \) of a finite group \( G \) is the Grothendieck group of finite \( G \)-sets, i.e.
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B(G) = \mathbb{Z}\{[G\text{-sets}]\}/\langle [X \sqcup Y] - [X] - [Y] \rangle.
\]

2. The group \( B(G) \) is a **free abelian group** on \( \{[G/H] \mid H \in [s_G]\} \).

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The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

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6. The prime spectrum of $B(G)$ can be explicitly described (Dress). $Spec B(G)$ connected
The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

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1. The **Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
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5. $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its **primitive idempotents** are known explicitly (Gluck 1981, Yoshida 1983).

6. The **prime spectrum** of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\iff G$ is solvable.
1. The **Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e. 
$$B(G) = \mathbb{Z} \langle [G\text{-sets}] \rangle / \langle [X \sqcup Y] - [X] - [Y] \rangle.$$

2. The group $B(G)$ is a **free abelian group** on $\{[G/H] | H \in [s_G]\}$.

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6. The **prime spectrum** of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\iff G$ is solvable.

7. The correspondence $G \mapsto B(G)$
The Burnside functor

1. The **Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
   \[ B(G) = \mathbb{Z}\{[G\text{-sets}]\} / < [X \sqcup Y] - [X] - [Y] >. \]
2. The group $B(G)$ is a **free abelian group** on $\{[G/H] \mid H \in [s_G]\}$.
3. The cartesian product of $G$-sets induces a **ring structure** on $B(G)$.
4. There is a **ghost ring homomorphism** $\Phi : B(G) \to \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside ≤ 1911), with finite explicit cokernel (Dress 1969).
5. $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its **primitive idempotents** are known explicitly (Gluck 1981, Yoshida 1983).
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   \[ \text{Spec } B(G) \text{ connected } \iff G \text{ is solvable.} \]
7. The correspondence $G \mapsto B(G)$ is a **Green biset functor**

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Serge Bouc (CNRS-LAMFA)
The Burnside functor

1. The **Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
   \[ B(G) = \mathbb{Z}\{[G\text{-sets}]\} / < [X \sqcup Y] - [X] - [Y] >. \]

2. The group $B(G)$ is a **free abelian group** on \{\([G/H] \mid H \in [s_G]\)\}.

3. The cartesian product of $G$-sets induces a **ring structure** on $B(G)$.

4. There is a **ghost ring homomorphism** $\Phi : B(G) \rightarrow \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).

5. $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its **primitive idempotents** are known explicitly (Gluck 1981, Yoshida 1983).

6. The **prime spectrum** of $B(G)$ can be explicitly described (Dress). $Spec B(G)$ connected $\iff G$ is solvable.

7. The correspondence $G \mapsto B(G)$ is a **Green biset functor** (there is a compatible product $B(G) \times B(G') \rightarrow B(G \times G')$)
The Burnside functor

1. The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
   $B(G) = \mathbb{Z}\{[G\text{-sets}]\} / \langle [X \sqcup Y] - [X] - [Y] \rangle$.
2. The group $B(G)$ is a free abelian group on $\{[G/H] \mid H \in [s_G]\}$.
3. The cartesian product of $G$-sets induces a ring structure on $B(G)$.
4. There is a ghost ring homomorphism $\Phi : B(G) \rightarrow \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).
5. $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).
6. The prime spectrum of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\iff G$ is solvable.
7. The correspondence $G \mapsto B(G)$ is a Green biset functor.
8. Tensor induction
The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

$$B(G) = \mathbb{Z}\{[G\text{-sets}]\}/<[X \sqcup Y] - [X] - [Y]>. $$

The group $B(G)$ is a free abelian group on $\{[G/H] \mid H \in [s_G]\}$. The cartesian product of $G$-sets induces a ring structure on $B(G)$. There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969). $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983). The prime spectrum of $B(G)$ can be explicitly described (Dress). $Spec B(G)$ connected $\iff G$ is solvable. The correspondence $G \mapsto B(G)$ is a Green biset functor. Tensor induction endows $G \mapsto B(G)^\times$ with a structure of biset functor.
The slice Burnside group

The slice Burnside group $\Xi(G)$ of a finite group $G$ is the Grothendieck group of morphisms of finite $G$-sets, i.e.

$$\Xi(G) = \mathbb{Z}\{G\text{-sets} \}/\langle X \sqcup Y \to Z \mid X \to f(X) \land Y \to f(Y) \rangle.$$
The slice Burnside group

1 The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

$$B(G) = \mathbb{Z}\{[G\text{-sets}]\} / \langle [X \sqcup Y] - [X] - [Y] \rangle.$$
The slice Burnside group

The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
$$B(G) = \mathbb{Z}\{[G\text{-sets}]\} / < [X \sqcup Y] - [X] - [Y] >.$$

Replace $G$-sets by the category $G\text{-Mor}$ of morphisms of $G$-sets:

→ Replace $G$-sets by the category $G\text{-Mor}$ of morphisms of $G$-sets:
The slice Burnside group

The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

$$B(G) = \mathbb{Z}\{[G\text{-sets}]\} / \langle [X \sqcup Y] - [X] - [Y] \rangle.$$ 

→ Replace $G$-sets by the category $G$-Mor of morphisms of $G$-sets: objects are morphisms $X \xrightarrow{f} Y$ of finite $G$-sets.
The slice Burnside group

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→ Replace $G$-sets by the category $G\text{-Mor}$ of morphisms of $G$-sets: objects are morphisms $X \xrightarrow{f} Y$ of finite $G$-sets, morphisms are pairs $(a, b)$ such that

\[
\begin{array}{ccc}
X & \xrightarrow{f} & Y \\
\downarrow{a} & & \downarrow{b} \\
Z & \xrightarrow{g} & T
\end{array}
\]

is commutative.
The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

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The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

$$B(G) = \mathbb{Z}^{\{G\text{-sets}\}} / < [X \sqcup Y] - [X] - [Y] >.$$

The slice Burnside group $\Xi(G)$ of a finite group $G$ is the Grothendieck group of morphisms of finite $G$-sets, i.e.

$$\Xi(G) = \mathbb{Z}^{\{X \rightarrow Z\}} / < X \sqcup Y \rightarrow Z - X \rightarrow \pi_1(X) - Y \rightarrow \pi_1(Y) >.$$

The group $B(G)$ is a free abelian group on $\{ [G/H] | H \in \mathbb{G}/s \}$. The group $\Xi(G)$ is a free abelian group on the set $\{ [G/S \rightarrow T] | (T, S) \in \Pi(G) \}$, where $\Pi(G)$ is the set of slices $(T, S)$ of $G$, i.e. pairs such that $S \leq T \leq G$, and $G/S \rightarrow G/T$ is the projection map.
The slice Burnside group

The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.

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The slice Burnside group $\Xi(G)$ of a finite group $G$ is the Grothendieck group of morphisms of finite $G$-sets.
The slice Burnside group

The Burnside group $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
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→ The **slice Burnside group** $\Xi(G)$ of a finite group $G$ is the Grothendieck group of *morphisms* of finite $G$-sets, i.e.

$$\Xi(G) = \mathbb{Z}\{[X \overset{f}{\to} Z]\} / < [X \sqcup Y \overset{f}{\to} Z] − [X \overset{f|_X}{\to} f(X)] − [Y \overset{f|_Y}{\to} f(Y)] >.$$
The slice Burnside group

1. The **Burnside group** $B(G)$ of a finite group $G$ is the Grothendieck group of finite $G$-sets, i.e.
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The slice Burnside group

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The slice Burnside ring

The cartesian product of $G$-sets induces a ring structure on $B(G)$.

The product of morphisms of $G$-sets induces a ring structure on $\Xi(G)$.

There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in \mathcal{S}_G} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).

This induces a ghost ring homomorphism $\Phi : \Xi(G) \to \prod_{(T, S) \in \Pi(G)} \mathbb{Z}$, injective, with finite explicit cokernel.
The cartesian product of $G$-sets induces a **ring structure** on $B(G)$. 

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There is a product in the category of morphisms of $G$-sets.
The cartesian product of $G$-sets induces a ring structure on $B(G)$.

There is a product in the category of morphisms of $G$-sets, defined by

$$(X \xrightarrow{f} Y) \times (Z \xrightarrow{g} T) = ((X \times Z) \xrightarrow{f \times g} (Y \times T)).$$
The slice Burnside ring

The cartesian product of $G$-sets induces a ring structure on $B(G)$. The product of morphisms of $G$-sets induces a ring structure on $\Xi(G)$. There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in \mathcal{S}_G} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969). This induces a ghost ring homomorphism $\Phi : \Xi(G) \to \prod_{(T, S) \in \mathcal{P}(\Sigma G)} \mathbb{Z}$, injective, with finite explicit cokernel.
The slice Burnside ring

3. The cartesian product of $G$-sets induces a ring structure on $B(G)$.

→ The product of morphisms of $G$-sets induces a ring structure on $\Xi(G)$.

4. There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in [s_G]} \mathbb{Z}$, induced by $\phi_H : X \mapsto |X^H|$.
The cartesian product of $G$-sets induces a ring structure on $B(G)$.

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There is a ghost ring homomorphism $\Phi : B(G) \to \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).
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3. The cartesian product of $G$-sets induces a **ring structure** on $B(G)$.

$\rightarrow$ The product of morphisms of $G$-sets induces a **ring structure** on $\Xi(G)$.

4. There is a **ghost ring homomorphism** $\Phi : B(G) \rightarrow \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).

$\rightarrow$ Let $(T, S) \in \Pi(G)$.
The slice Burnside ring

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Let $(T, S) \in \Pi(G)$. For a morphism $m = (X \xrightarrow{f} Y)$, define

$$\phi_{T,S}(m) =$$
The cartesian product of $G$-sets induces a ring structure on $B(G)$.

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There is a ghost ring homomorphism $\Phi : B(G) \rightarrow \prod_{H \in [s_G]} \mathbb{Z}$, injective (Burnside $\leq 1911$), with finite explicit cokernel (Dress 1969).

Let $(T, S) \in \Pi(G)$. For a morphism $m = (X \xrightarrow{f} Y)$, define $\phi_{T,S}(m) = \text{number of morphisms}
\begin{pmatrix}
  G/S & G/T \\
  a & b \\
  X & Y
\end{pmatrix}.

The slice Burnside ring

3 The cartesian product of $G$-sets induces a ring structure on $B(G)$.

→ The product of morphisms of $G$-sets induces a ring structure on $\Xi(G)$.

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→ Let $(T, S) \in \Pi(G)$. For a morphism $m = (X \xrightarrow{f} Y)$, define

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\end{pmatrix}.$$

$$\phi_{T, S}(m) = |f^{-1}(Y^T)^S|.$$  
This induces a ring homomorphism $\phi_{T, S} : \Xi(G) \to \mathbb{Z}$. 
The slice Burnside ring

3. The cartesian product of $G$-sets induces a **ring structure** on $B(G)$.

→ The product of morphisms of $G$-sets induces a **ring structure** on $\Xi(G)$.

4. There is a **ghost ring homomorphism** $\Phi : B(G) \to \prod_{H \in [s_G]} \mathbb{Z}$, injective (**Burnside $\leq 1911$**), with finite explicit cokernel (**Dress 1969**).

→ This induces a **ghost ring homomorphism**
The slice Burnside ring

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$$\Phi : \Xi(G) \to \prod_{(T,S) \in \mathcal{P}(G)} \mathbb{Z}$$
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The slice Burnside ring

- The slice Burnside ring $B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

- $\mathbb{Q}\mathfrak{Ξ}(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents $\mathfrak{Ξ}_G(T), \mathfrak{Ξ}_S$ are known explicitly.

- The prime spectrum of $B(G)$ can be explicitly described (Dress). Spec $B(G)$ connected $\iff G$ is solvable.

- The prime spectrum of $\mathbb{Q}\mathfrak{Ξ}(G)$ can be explicitly described. Spec $\mathbb{Q}\mathfrak{Ξ}(G)$ connected $\iff G$ is solvable.
$\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).
The slice Burnside ring

\[ \mathbb{Q}B(G) \] is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

\[ \mathbb{Q} \Xi(G) \] is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi_G^T, S \) are indexed by slices of \( G \) up to conjugation.

\[ \xi_G^T, S = \frac{1}{|N_G(T, S)|} \sum_{U \leq S \leq V \leq T} |U| \mu(U, S) \mu(V, T) [G/U \to G/V]. \]

\[ \mathbb{Q} \Xi(G) \] is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents are known explicitly.

The prime spectrum of \( B(G) \) can be explicitly described (Dress).

\[ \text{Spec } B(G) \text{ connected } \iff G \text{ is solvable}. \]

The prime spectrum of \( \Xi(G) \) can be explicitly described.

\[ \text{Spec } \Xi(G) \text{ connected } \iff G \text{ is solvable}. \]
The slice Burnside ring

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\( \text{Spec } B(G) \text{ connected } \iff G \text{ is solvable.} \)

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QB(G) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its **primitive idempotents** are known explicitly (Gluck 1981, Yoshida 1983).

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\[
\xi_{T,S}^G = \frac{1}{|N_G(T, S)|} \sum_{U \leq S \leq V \leq T} |U| \mu(U, S) \mu(V, T) [G/U \to G/V].
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\( \mathbb{Q} \Xi(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi^G_T, S \) are known explicitly.
The slice Burnside ring

5. \( \mathbb{Q}B(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

→ \( \mathbb{Q}\Xi(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi^G_T, S \) are known explicitly.

6. The prime spectrum of \( B(G) \) can be explicitly described (Dress). Spec \( B(G) \) connected \( \iff \) \( G \) is solvable.
The slice Burnside ring

5 $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

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Let $\Theta(G) = \{ (T, S, p) \mid (T, S) \in \Pi(G) \}$
The slice Burnside ring

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5. \(\mathbb{Q}B(G)\) is a split semisimple commutative \(\mathbb{Q}\)-algebra. Its \textbf{primitive idempotents} are known explicitly (Gluck 1981, Yoshida 1983).

\(\rightarrow\) \(\mathbb{Q}\Xi(G)\) is a split semisimple commutative \(\mathbb{Q}\)-algebra. Its \textbf{primitive idempotents} \(\xi_T^G, S\) are known explicitly.

6. The \textbf{prime spectrum} of \(B(G)\) can be explicitly described (Dress). \(\text{Spec } B(G)\) connected \(\iff G\) is solvable.

Let \(\Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \notin p\} \).
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The slice Burnside ring

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Let $\Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \notin p\}$. For $(T, S, p) \in \Theta(G)$, consider $\Xi(G) \xrightarrow{\phi_{T,S}} \mathbb{Z} \to \mathbb{Z}/p$. 
The slice Burnside ring

**5** \( \mathbb{Q}B(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its **primitive idempotents** are known explicitly (Gluck 1981, Yoshida 1983).

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**6** The **prime spectrum** of \( B(G) \) can be explicitly described (Dress). \( \text{Spec } B(G) \text{ connected } \iff G \text{ is solvable.} \)

Let \( \Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \notin p \} \).

For \( (T, S, p) \in \Theta(G) \), consider \( \ker(\Xi(G)^{\phi_{T,S}} \mathbb{Z} \to \mathbb{Z}/p) \).
The slice Burnside ring

\[ \mathbb{Q}B(G) \] is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

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Let \( \Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \notin p\} \).

For \( (T, S, p) \in \Theta(G) \), set \( I_{T,S,p} = \text{Ker}(\Xi(G)^{\phi_{T,S}} : \mathbb{Z} \to \mathbb{Z}/p) \).
The slice Burnside ring

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\( \mathbb{Q}\Xi(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi^G_{T,S} \) are known explicitly.

The prime spectrum of \( B(G) \) can be explicitly described (Dress). \( \text{Spec } B(G) \) connected \iff \( G \) is solvable.

Let \( \Theta(G) = \{ (T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, \left| N_G(T, S) : S \right| \notin p \} \).

For \( (T, S, p) \in \Theta(G) \), set \( I_{T,S,p} = \text{Ker}(\Xi(G) \xrightarrow{\phi_{T,S}} \mathbb{Z} \to \mathbb{Z}/p) \).

Then \( \text{Spec } \Xi(G) = \{ I_{T,S,p} \mid (T, S, p) \in [\Theta(G)] \} \).
\( \mathbb{Q}B(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

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When \( p \) is a prime
The slice Burnside ring

5 $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

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6 The prime spectrum of $B(G)$ can be explicitly described (Dress).

$\text{Spec } B(G)$ connected $\iff G$ is solvable.

Let $\Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |NG(T, S) : S| \notin p\}$.

For $(T, S, p) \in \Theta(G)$, set $I_{T,S,p} = \text{Ker}(\Xi(G) \xrightarrow{\phi_{T,S}} \mathbb{Z} \rightarrow \mathbb{Z}/p)$.

Then $\text{Spec } \Xi(G) = \{I_{T,S,p} \mid (T, S, p) \in [\Theta(G)]\}$.

When $p$ is a prime, and $(T, S) \in \Pi(G)$
The slice Burnside ring

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The prime spectrum of \( B(G) \) can be explicitly described (Dress).

\[ \text{Spec } B(G) \text{ connected } \iff G \text{ is solvable}. \]

Let \( \Theta(G) = \{ (T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \notin p \}. \)

For \( (T, S, p) \in \Theta(G) \), set \( I_{T, S, p} = \ker(\Xi(G) \xrightarrow{\phi_{T, S}} \mathbb{Z} \to \mathbb{Z}/p) \).

Then \( \text{Spec } \Xi(G) = \{ I_{T, S, p} \mid (T, S, p) \in [\Theta(G)] \} \).

When \( p \) is a prime, and \( (T, S) \in \Pi(G) \), define \( (T, S)^+_p = (PT, PS) \).
The slice Burnside ring

1. \( \mathbb{Q}B(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

2. \( \mathbb{Q}\Xi(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi_T^G \) are known explicitly.

3. The prime spectrum of \( B(G) \) can be explicitly described (Dress).

   \[ Spec \ B(G) \text{ connected } \iff G \text{ is solvable.} \]

Let \( \Theta(G) = \{ (T, S, p) \mid (T, S) \in \Pi(G), p \in Spec \mathbb{Z}, |N_G(T, S) : S| \notin p \} \).

For \( (T, S, p) \in \Theta(G) \), set \( I_{T,S,p} = \text{Ker}(\Xi(G) \xrightarrow{\phi_T^S} \mathbb{Z} \to \mathbb{Z}/p) \).

Then \( Spec \ \Xi(G) = \{ I_{T,S,p} \mid (T, S, p) \in [\Theta(G)] \} \).

When \( p \) is a prime, and \( (T, S) \in \Pi(G) \), define \( (T, S)^+_p = (PT, PS) \),

where \( P \in Syl_p(N_G(T, S)) \).
The slice Burnside ring

$\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

→ $\mathbb{Q}\Xi(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents $\xi^G_T, S$ are known explicitly.

The prime spectrum of $B(G)$ can be explicitly described (Dress).

$\text{Spec } B(G)$ connected $\iff G$ is solvable.

Let $\Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \notin p\}$.

For $(T, S, p) \in \Theta(G)$, set $I_{T,S,p} = \ker(\Xi(G) \phi_{T,S} : \mathbb{Z} \rightarrow \mathbb{Z}/p)$.

→ Then $\text{Spec } \Xi(G) = \{I_{T,S,p} \mid (T, S, p) \in [\Theta(G)]\}$.

When $p$ is a prime, and $(T, S) \in \Pi(G)$, define $(T, S)^+_p = (PT, PS)$, where $P \in \text{Syl}_p(N_G(T, S))$, and $(T, S)^+_p = \lim(((T, S)^+_p)_p^+)$. ...
The slice Burnside ring

1. $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

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2. The prime spectrum of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\iff G$ is solvable.

Let $\Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \not\in p\}.$

For $(T, S, p) \in \Theta(G)$, set $I_{T, S, p} = \text{Ker}(\Xi(G) \xrightarrow{\phi_{T, S}} \mathbb{Z} \to \mathbb{Z}/p)$.

Then $\text{Spec } \Xi(G) = \{I_{T, S, p} \mid (T, S, p) \in [\Theta(G)]\}.$

If $(T, S, p), (V, U, q) \in \Theta(G)$
The slice Burnside ring

QB(G) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its **primitive idempotents** are known explicitly (Gluck 1981, Yoshida 1983).

\[ \mathbb{Q} \Xi(G) \] is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its **primitive idempotents** \( \xi_{T,S}^G \) are known explicitly.

The **prime spectrum** of \( B(G) \) can be explicitly described (Dress).

**Spec** \( B(G) \) connected \( \iff \) \( G \) is solvable.

Let \( \Theta(G) = \{ (T, S, p) \mid (T, S) \in \Pi(G), p \in Spec \mathbb{Z}, |N_G(T, S) : S| \notin p \} \).

For \( (T, S, p) \in \Theta(G) \), set \( I_{T,S,p} = \text{Ker}(\Xi(G)^{\phi_{T,S}} : \mathbb{Z} \to \mathbb{Z}/p) \).

Then **Spec** \( \Xi(G) = \{ I_{T,S,p} \mid (T, S, p) \in [\Theta(G)] \} \).

If \( (T, S, p), (V, U, q) \in \Theta(G) \), then \( I_{T,S,p} \subseteq I_{V,U,q} \) if and only if
The slice Burnside ring

5 $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

$\rightarrow \mathbb{Q}\Xi(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents $\xi^G_T, S$ are known explicitly.

6 The prime spectrum of $B(G)$ can be explicitly described (Dress).

Spec $B(G)$ connected $\iff G$ is solvable.

Let $\Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in Spec \mathbb{Z}, |N_G(T, S) : S| \notin p\}.$

For $(T, S, p) \in \Theta(G)$, set $I_{T, S, p} = \text{Ker}(\Xi(G) \phi_{T, S} : \mathbb{Z} \to \mathbb{Z}/p).$

$\rightarrow$ Then Spec $\Xi(G) = \{I_{T, S, p} \mid (T, S, p) \in \Theta(G)\}.$

If $(T, S, p), (V, U, q) \in \Theta(G),$ then $I_{T, S, p} \subseteq I_{V, U, q}$ if and only if $p = q$ and $(T, S) \equiv_G (V, U)$
The slice Burnside ring

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\( \rightarrow \) \( \mathbb{Q}\Xi(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi^G_T,S \) are known explicitly.

6 The prime spectrum of \( B(G) \) can be explicitly described (Dress).

\( \text{Spec } B(G) \) connected \( \iff \) \( G \) is solvable.

Let \( \Theta(G) = \{ (T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S): S| \notin p \} \).

For \((T, S, p) \in \Theta(G)\), set \( I_{T,S,p} = \ker(\Xi(G)^{\phi_{T,S}} \mathbb{Z} \to \mathbb{Z}/p)\).

Then \( \text{Spec } \Xi(G) = \{ I_{T,S,p} \mid (T, S, p) \in [\Theta(G)] \} \).

If \((T, S, p), (V, U, q) \in \Theta(G)\), then \( I_{T,S,p} \subseteq I_{V,U,q} \) if and only if

- \( p = q \) and \((T, S) =_G (V, U)\)
- \( p = \{0\} \) and \( q = q\mathbb{Z} \) and \((T, S)\hat{q} =_G (V, U)\)
The slice Burnside ring

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\[ \rightarrow \mathbb{Q}\Xi(G) \] is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi^G_T,S \) are known explicitly.

6 The prime spectrum of \( B(G) \) can be explicitly described (Dress). \( \text{Spec } B(G) \) connected \( \iff \) \( G \) is solvable.

Let \( \Theta(G) = \{(T, S, p) \mid (T, S) \in \Pi(G), p \in \text{Spec } \mathbb{Z}, |N_G(T, S) : S| \not\in p\} \).

For \((T, S, p) \in \Theta(G)\), set \( I_{T,S,p} = \text{Ker}(\Xi(G) \xrightarrow{\phi_{T,S}} \mathbb{Z} \rightarrow \mathbb{Z}/p) \).

\[ \rightarrow \text{Then } \text{Spec } \Xi(G) = \{I_{T,S,p} \mid (T, S, p) \in [\Theta(G)]\}. \]

If \((T, S, p), (V, U, q) \in \Theta(G)\), then \( I_{T,S,p} \subseteq I_{V,U,q} \) if and only if

\[ \begin{array}{l}
p = q \text{ and } (T, S) = G (V, U) \\
p = \{0\} \text{ and } q = q\mathbb{Z} \text{ and } (T, S)^q = G (V, U) \end{array} \]
5 $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

$\rightarrow$ $\mathbb{Q}\Xi(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents $\xi^G_T, S$ are known explicitly.

6 The prime spectrum of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\iff$ $G$ is solvable.

$\rightarrow$ The prime spectrum of $\Xi(G)$ can be explicitly described.
The slice Burnside ring

5 $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

$\mathbb{Q}\Xi(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents $\xi^G_T, S$ are known explicitly.

6 The prime spectrum of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\Leftrightarrow$ $G$ is solvable.

$\text{Spec } \Xi(G)$ connected

$\text{Spec } \Xi(G)$ connected
The slice Burnside ring

5. \( \mathbb{Q}B(G) \) is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents are known explicitly (Gluck 1981, Yoshida 1983).

\[ \mathbb{Q} \bar{\Xi}(G) \] is a split semisimple commutative \( \mathbb{Q} \)-algebra. Its primitive idempotents \( \xi_{T,S}^G \) are known explicitly.

6. The prime spectrum of \( B(G) \) can be explicitly described (Dress). \( \text{Spec } B(G) \) connected \( \iff \) \( G \) is solvable.

\[ \text{Spec } \bar{\Xi}(G) \] can be explicitly described. \( \text{Spec } \bar{\Xi}(G) \) connected \( \iff \) \( G \) is solvable.
The slice Burnside ring

5 $\mathbb{Q}B(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its **primitive idempotents** are known explicitly (Gluck 1981, Yoshida 1983).

→ $\mathbb{Q}\Xi(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its **primitive idempotents** $\xi^G_T,S$ are known explicitly.

6 The **prime spectrum** of $B(G)$ can be explicitly described (Dress). $\text{Spec } B(G)$ connected $\iff G$ is solvable.

→ The **prime spectrum** of $\Xi(G)$ can be explicitly described. $\text{Spec } \Xi(G)$ connected $\iff G$ is solvable.
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Tensor induction endows $G \mapsto B(G)$ with a structure of biset functor.

The correspondence $G \mapsto \Xi(G)$ cannot be endowed with a structure of biset functor.
The correspondence $G \mapsto B(G)$ is a Green biset functor.
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Let $G$ and $H$ be finite groups.

Tensor induction endows $G \mapsto B(G)$ with a structure of biset functor.

The correspondence $G \mapsto \Xi(G)$ cannot be endowed with a structure of biset functor.
The correspondence \( G \mapsto B(G) \) is a **Green biset functor**.

Let \( G \) and \( H \) be finite groups, and \( U \) be a finite \((H, G)\)-biset.
The correspondence $G \mapsto B(G)$ is a **Green biset functor**.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_{G} X \xrightarrow{U \times_{G} f} U \times_{G} Y)$
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \to \Xi(H)$. Tensor induction endows $G \mapsto B(G) \times$ with a structure of biset functor. The correspondence $G \mapsto \Xi(G) \times$ cannot be endowed with a structure of biset functor.
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \overset{f}{\rightarrow} Y) \mapsto (U \times G X \overset{U \times Gf}{\rightarrow} U \times G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \rightarrow \Xi(H)$. Hence $\Xi$ is a biset functor.
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \rightarrow \Xi(H)$. Hence $\Xi$ is a biset functor.

Let $G, G'$ be finite groups.
The correspondence \( G \mapsto B(G) \) is a Green biset functor.

Let \( G \) and \( H \) be finite groups, and \( U \) be a finite \((H, G)\)-biset. The functor \((X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)\) induces a group homomorphism \( \Xi(U) : \Xi(G) \to \Xi(H) \). Hence \( \Xi \) is a biset functor.

Let \( G, G' \) be finite groups. The functor \((X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}\).
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \rightarrow Y) \mapsto (U \times_G X \xrightarrow{U \times G f} U \times_G Y)$ induces a group homomorphism $\Theta(U) : \Theta(G) \rightarrow \Theta(H)$. Hence $\Theta$ is a biset functor.

Let $G, G'$ be finite groups. The functor

$$(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}$$

induces a product $\Theta(G) \times \Theta(G') \rightarrow \Theta(G \times G')$. Tensor induction endows $G \mapsto B(G) \times$ with a structure of biset functor.

The correspondence $G \mapsto \Theta(G)$ cannot be endowed with a structure of biset functor.
Let \( G \) and \( H \) be finite groups, and \( U \) be a finite \((H, G)\)-biset. The functor \((X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)\) induces a group homomorphism \( \Xi(U) : \Xi(G) \to \Xi(H) \). Hence \( \Xi \) is a biset functor.

Let \( G, G' \) be finite groups. The functor

\[
(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}
\]

induces a product \( \Xi(G) \times \Xi(G') \to \Xi(G \times G') \), associative
The correspondence $G \mapsto B(G)$ is a **Green biset functor**.

→ Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \to \Xi(H)$. Hence $\Xi$ is a **biset functor**.

→ Let $G, G'$ be finite groups. The functor

$$(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}$$

induces a product $\Xi(G) \times \Xi(G') \to \Xi(G \times G')$, associative, unital
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times G X \xrightarrow{U \times G f} U \times G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \to \Xi(H)$. Hence $\Xi$ is a biset functor.

Let $G, G'$ be finite groups. The functor

$$(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}$$

induces a product $\Xi(G) \times \Xi(G') \to \Xi(G \times G')$, associative, unital, and compatible with the biset functor structure.
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \to \Xi(H)$. Hence $\Xi$ is a biset functor.

Let $G, G'$ be finite groups. The functor
\[(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}\]
induces a product $\Xi(G) \times \Xi(G') \to \Xi(G \times G')$, associative, unital, and compatible with the biset functor structure: the diagram
\[
\begin{array}{ccc}
\Xi(G) \times \Xi(G') & \longrightarrow & \Xi(G \times G') \\
\downarrow & & \downarrow \\
\Xi(H) \times \Xi(H') & \longrightarrow & \Xi(H \times H')
\end{array}
\]
is commutative.

Tensor induction endows $G \mapsto B(G)$ with a structure of biset functor.

The correspondence $G \mapsto \Xi(G)$ cannot be endowed with a structure of biset functor.
The slice Burnside functor

The correspondence $G \mapsto B(G)$ is a Green biset functor.

→ Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times G f} U \times_G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \to \Xi(H)$. Hence $\Xi$ is a biset functor.

→ Let $G, G'$ be finite groups. The functor

$$(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}$$

induces a product $\Xi(G) \times \Xi(G') \to \Xi(G \times G')$, associative, unital, and compatible with the biset functor structure: the diagram

$$
\begin{array}{ccc}
\Xi(G) & \times & \Xi(G') \\
\downarrow \Xi(U) & & \downarrow \Xi(U') \\
\Xi(H) & \times & \Xi(H') \\
\end{array}
\xrightarrow{\Xi(U \times U')}
\Xi(G \times G')
$$

is commutative, for any finite groups $H$ and $H'$. Tensor induction endows $G \mapsto B(G)$ with a structure of biset functor.

The correspondence $G \mapsto \Xi(G)$ cannot be endowed with a structure of biset functor.
The correspondence $G \mapsto B(G)$ is a Green biset functor.

Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \to \Xi(H)$. Hence $\Xi$ is a biset functor.

Let $G, G'$ be finite groups. The functor $(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}$ induces a product $\Xi(G) \times \Xi(G') \to \Xi(G \times G')$, associative, unital, and compatible with the biset functor structure: the diagram

\[
\begin{array}{ccc}
\Xi(G) & \times & \Xi(G') \\
\downarrow & & \downarrow \Xi(U') \\
\Xi(H) & \times & \Xi(H')
\end{array} \quad \Xi(U) \quad \Xi(U \times U') \quad \Xi(H \times H')
\]

is commutative, for any finite groups $H$ and $H'$, any finite $(H, G)$-biset $U$. 

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The slice Burnside functor

The correspondence $G \mapsto B(G)$ is a Green biset functor.

→ Let $G$ and $H$ be finite groups, and $U$ be a finite $(H, G)$-biset. The functor $(X \xrightarrow{f} Y) \mapsto (U \times_G X \xrightarrow{U \times_G f} U \times_G Y)$ induces a group homomorphism $\Xi(U) : \Xi(G) \rightarrow \Xi(H)$. Hence $\Xi$ is a biset functor.

→ Let $G, G'$ be finite groups. The functor $(X, X') \in G\text{-set} \times G'\text{-set} \mapsto (X \times X') \in (G \times G')\text{-set}$ induces a product $\Xi(G) \times \Xi(G') \rightarrow \Xi(G \times G')$, associative, unital, and compatible with the biset functor structure: the diagram

$$
\begin{array}{ccc}
\Xi(G) & \times & \Xi(G') \\
\downarrow \Xi(U) & & \downarrow \Xi(U') \\
\Xi(H) & \times & \Xi(H')
\end{array}
\rightarrow
\begin{array}{c}
\Xi(G \times G') \\
\Xi(U \times U')
\end{array}
$$

is commutative, for any finite groups $H$ and $H'$, any finite $(H, G)$-biset $U$, any finite $(H', G')$-biset $U'$.
The correspondence $G \mapsto B(G)$ is a Green biset functor.
The slice Burnside functor

The correspondence $G \mapsto B(G)$ is a Green biset functor.

The correspondence $G \mapsto \Xi(G)$ is a Green biset functor.
The slice Burnside functor

7. The correspondence $G \mapsto B(G)$ is a Green biset functor.

8. Tensor induction endows $G \mapsto B(G)^\times$ with a structure of biset functor.

→ The correspondence $G \mapsto \Xi(G)$ is a Green biset functor.
The slice Burnside functor

7. The correspondence $G \mapsto B(G)$ is a Green biset functor.

8. Tensor induction endows $G \mapsto B(G)^\times$ with a structure of biset functor.

→ Suppose that there is such a biset functor $\Xi^\times$.
The slice Burnside functor

The correspondence \( G \mapsto B(G) \) is a Green biset functor.

The correspondence \( G \mapsto \Xi(G) \) is a Green biset functor.

Tensor induction endows \( G \mapsto B(G)^\times \) with a structure of biset functor.

Suppose that there is such a biset functor \( \Xi^\times(\Xi^\times(G) = \Xi(G)^\times) \).

If \( G \) is abelian, then \( \dim_{F_2} \Xi(G)^\times = 2r + 1 \), where \( r \) is the number of subgroups of index 2 in \( G \). Hence \( \dim_{F_2} \Xi^\times(\Xi^\times(C_2)^2) = 7 < 9 \).
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→ Easy computations of $\Xi(G)^\times$ in the case $G = 1$, $G = C_2$. 

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The slice Burnside functor

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If $G$ is abelian, then $\dim_{\mathbb{F}_2} \Xi(G)^\times = 2r + 1$, where $r$ is the number of subgroups of index 2 in $G$. Hence $\dim_{\mathbb{F}_2} \Xi^\times((C_2)^2) = 7$. 

If $G$ is abelian, then $\dim_{\mathbb{F}_2} \Xi(G)^\times$ cannot be endowed with a structure of biset functor.
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Galois morphisms of $G$-sets

**Definition**

Let $G$ be a group. A morphism of $G$-sets $f : X \to Y$ is called a Galois morphism if

$$\forall x, x' \in X, \quad f(x) = f(x') \Rightarrow \exists \phi \in \text{Aut}_G\text{-Set}(X), \quad \phi(x) = x' \text{ and } f \circ \phi = f.$$ 

Let $G\text{-Mor}\text{Gal}$ denote the full subcategory of $G\text{-Mor}$ consisting of Galois morphisms of $G$-sets.

**Lemma**

Let $f : X \to Y$ be a morphism of $G$-sets. The following are equivalent:

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**Example**: Let $(T, S) \in \Pi(G)$. Then:
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**Definition**

A section $(T, S)$ of $G$ is a slice such that $S \sqsubseteq T$. Let $\Sigma(G)$ denote the set of sections of $G$. 

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Properties of Galois morphisms

Let \( f: X \to Y \) be a Galois morphism of \( G \)-sets, and \( X_1 \subseteq X \) be a \( G \)-subset of \( X \).

Then \( f|_{X_1}: X_1 \to f(X_1) \) is a Galois morphism of \( G \)-sets.

Let \( f: X \to Y \) and \( g: Z \to T \) be Galois morphisms of \( G \)-sets. Then \( f \times g: X \times Z \to Y \times T \) is a Galois morphism of \( G \)-sets.

Let \( f: X \to Y \) be a Galois morphism of \( G \)-sets, and \( U \) be an \((H, G)\)-biset.

Then \( U \times_G f: U \times_G X \to U \times_G Y \) is a Galois morphism of \( H \)-sets.

The correspondence \((f: X \to Y) \mapsto (f_{\text{Gal}}: X_{\text{Gal}} f \to Y)\) is a functor \( G\text{-Mor} \to G\text{-Mor} \text{Gal} \), left adjoint to the forgetful functor \( G\text{-Mor} \text{Gal} \to G\text{-Mor} \).

Thus \( G\text{-Mor} \text{Gal} \) is a reflective subcategory of \( G\text{-Mor} \).

Example: Let \((T, S) \in \Pi(G)\), and \( f: G/S \to G/T \) be the projection map.

Then \( f_{\text{Gal}} \) is the projection map \( G/S \triangleright T \to G/T \), where \( S \triangleright T \) is the normal closure of \( S \) in \( T \).
Let \( f : X \rightarrow Y \) be a Galois morphism of \( G \)-sets.

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Then \( f \times g : X \times Z \rightarrow Y \times T \) is a Galois morphism of \( G \)-sets.

Let \( f : X \rightarrow Y \) be a Galois morphism of \( G \)-sets, and \( U \) be an \((H, G)\)-biset.

Then \( U \times G f : U \times G X \rightarrow U \times G Y \) is a Galois morphism of \( H \)-sets.

The correspondence \( (f : X \rightarrow Y) \mapsto (f_{\text{Gal}} : X_{\text{Gal}} f \rightarrow Y) \) is a functor \( G\text{-Mor} \rightarrow G\text{-Mor}_{\text{Gal}} \), left adjoint to the forgetful functor \( G\text{-Mor}_{\text{Gal}} \rightarrow G\text{-Mor}. \)

Thus \( G\text{-Mor}_{\text{Gal}} \) is a reflective subcategory of \( G\text{-Mor}. \)

Example: Let \((T, S) \in \Pi(G)\), and \( f : G/S \rightarrow G/T \) be the projection map.

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- Let $f : X \rightarrow Y$ be any morphism of $G$-sets.
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- Let $f : X \rightarrow Y$ be any morphism of $G$-sets.
  - For $x \in X$, set $G_x^f = \langle G_z | z \in f^{-1}f(x) \rangle$. 

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Properties of Galois morphisms

- Let $f : X \to Y$ be a Galois morphism of $G$-sets, and $X_1 \subseteq X$ be a $G$-subset of $X$. Then $f|_{X_1} : X_1 \to f(X_1)$ is a Galois morphism of $G$-sets.

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  - For $x, x' \in X$, set $x \sim_f x'$ if there exists $g \in G_x^f$ such that $gx = x'$.
  - Then $\sim_f$ is a $G$-equivariant equivalence relation on $X$. 
Properties of Galois morphisms

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  - Then $\sim_f$ is a $G$-equivariant equivalence relation on $X$. The projection map $\gamma_{X,f} : X \to X_f^{Gal} = X/\sim_f$ is a map of $G$-sets.
Properties of Galois morphisms

Let $f : X \to Y$ be a Galois morphism of $G$-sets, and $X_1 \subseteq X$ be a $G$-subset of $X$. Then $f_{|X_1} : X_1 \to f(X_1)$ is a Galois morphism of $G$-sets.

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Let $f : X \to Y$ be a Galois morphism of $G$-sets, and $U$ be an $(H, G)$-biset. Then $U \times_G f : U \times_G X \to U \times_G Y$ is a Galois morphism of $H$-sets.

Let $f : X \to Y$ be any morphism of $G$-sets.

- For $x \in X$, set $G^f_x = \langle G_z \mid z \in f^{-1}f(x) \rangle$.
- For $x, x' \in X$, set $x \sim_f x'$ if there exists $g \in G^f_x$ such that $gx = x'$.
- Then $\sim_f$ is a $G$-equivariant equivalence relation on $X$. The projection map $\gamma_{X,f} : X \to X^f_{Gal} = X/\sim_f$ is a map of $G$-sets.
- There is a unique map of $G$-sets $f^Gal : X^f_{Gal} \to Y$ such that

$$
\begin{array}{ccc}
X & \xrightarrow{\gamma_{X,f}} & X^f_{Gal} \\
& & \xrightarrow{f^Gal} \\
& & Y
\end{array}
$$
Properties of Galois morphisms

- Let \( f : X \rightarrow Y \) be a Galois morphism of \( G \)-sets, and \( X_1 \subseteq X \) be a \( G \)-subset of \( X \). Then \( f|_{X_1} : X_1 \rightarrow f(X_1) \) is a Galois morphism of \( G \)-sets.

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  - For \( x \in X \), set \( G^f_x = \langle G_z \mid z \in f^{-1}f(x) \rangle \).
  - For \( x, x' \in X \), set \( x \sim_f x' \) if there exists \( g \in G^f_x \) such that \( gx = x' \).
  - Then \( \sim_f \) is a \( G \)-equivariant equivalence relation on \( X \). The projection map \( \gamma_{X, f} : X \rightarrow X^f_{Gal} = X/\sim_f \) is a map of \( G \)-sets.
  - There is a unique map of \( G \)-sets \( f^f_{Gal} : X^f_{Gal} \rightarrow Y \) such that

\[
X \xrightarrow{\gamma_{X, f}} X^f_{Gal} \xrightarrow{f^f_{Gal}} Y.
\]

Moreover \( f^f_{Gal} \) is a Galois morphism.
Properties of Galois morphisms

- Let \( f : X \to Y \) be a Galois morphism of \( G \)-sets, and \( X_1 \subseteq X \) be a \( G \)-subset of \( X \). Then \( f_{|X_1} : X_1 \to f(X_1) \) is a Galois morphism of \( G \)-sets.

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- The correspondence \((f : X \to Y) \mapsto (f^{Gal} : X_f^{Gal} \to Y)\) is a functor \( G\text{-Mor} \to G\text{-Mor}^{Gal} \).
Properties of Galois morphisms

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Properties of Galois morphisms

- Let $f : X \to Y$ be a Galois morphism of $G$-sets, and $X_1 \subseteq X$ be a $G$-subset of $X$. Then $f_\mid_{X_1} : X_1 \to f(X_1)$ is a Galois morphism of $G$-sets.

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- The correspondence $(f : X \to Y) \mapsto (f^{\text{Gal}} : X^{\text{Gal}}_f \to Y)$ is a functor $G\text{-Mor} \to G\text{-Mor}^{\text{Gal}}$, left adjoint to the forgetful functor $G\text{-Mor}^{\text{Gal}} \to G\text{-Mor}$. Thus $G\text{-Mor}^{\text{Gal}}$ is a reflective subcategory of $G\text{-Mor}$. 

Example: Let $(T, S) \in \Pi(G)$, and $f : G/\!\!/S \to G/\!\!/T$ be the projection map. Then $f^{\text{Gal}}$ is the projection map $G/\!\!/S \trianglelefteq T \to G/\!\!/T$, where $S \trianglelefteq T$ is the normal closure of $S$ in $T$. 

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Properties of Galois morphisms

- Let \( f : X \rightarrow Y \) be a Galois morphism of \( G \)-sets, and \( X_1 \subseteq X \) be a \( G \)-subset of \( X \). Then \( f|_{X_1} : X_1 \rightarrow f(X_1) \) is a Galois morphism of \( G \)-sets.

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**Example:** Let \((T, S) \in \Pi(G)\)
Properties of Galois morphisms

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- The correspondence $(f : X \to Y) \leftrightarrow (f^{Gal} : X^{f^{Gal}} \to Y)$ is a functor $G$-Mor $\to G$-Mor$^{Gal}$, left adjoint to the forgetful functor $G$-Mor$^{Gal} \to G$-Mor. Thus $G$-Mor$^{Gal}$ is a reflective subcategory of $G$-Mor.

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- Let $f : X \to Y$ be a Galois morphism of $G$-sets, and $X_1 \subseteq X$ be a $G$-subset of $X$. Then $f_{|X_1} : X_1 \to f(X_1)$ is a Galois morphism of $G$-sets.

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- **Example**: Let $(T, S) \in \Pi(G)$, and $f : G/S \to G/T$ be the projection map. Then $f^{Gal}$ is the projection map $G/S^{\triangleleft T} \to G/T$.
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The section Burnside ring

1. The Burnside group $\Gamma(G)$ of a finite group $G$ is the Grothendieck group of $G$-Mor, or equivalently the subgroup of $\Xi(G)$ generated by Galois morphisms of $G$-sets.

2. $\Gamma(G)$ is a free abelian group on the set of $[G/S \to G/T]$ for $(T, S) \in \Sigma(G)$.

3. $\Gamma(G)$ is a subring of $\Xi(G)$.

4. There is a ghost ring homomorphism $\Psi : \Gamma(G) \to \prod_{H \in \Sigma(G)} \mathbb{Z}$, injective, with finite explicit cokernel.

5. $\mathbb{Q}[\Gamma(G)]$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its primitive idempotents are known explicitly.

6. The prime spectrum of $\Gamma(G)$ can be explicitly described. $\text{Spec} \Gamma(G)$ connected $\iff G$ is solvable.

7. The correspondence $G \mapsto \Gamma(G)$ is a Green biset functor.

8. The correspondence $G \mapsto \Gamma(G)$ cannot be endowed with a structure of biset functor.
The section Burnside ring

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The section Burnside ring

1. The **section Burnside group** $\Gamma(G)$ of a finite group $G$ is the Grothendieck group of $G\text{-Mor}^{\text{Gal}}$, or equivalently the subgroup of $\Xi(G)$ generated by Galois morphisms of $G$-sets.

2. The group $\Gamma(G)$ is a **free abelian group** on the set $
\{ [G/S \to G/T] \mid (T, S) \in [\Sigma(G)] \}.
$

3. $\Gamma(G)$ is a **subring** of $\Xi(G)$.

4. There is a **ghost ring homomorphism** $\Psi : \Gamma(G) \to \prod_{H \in [\Sigma(G)]} \mathbb{Z}$, injective, with finite explicit cokernel.

5. $\mathbb{Q}\Gamma(G)$ is a split semisimple commutative $\mathbb{Q}$-algebra. Its **primitive idempotents** are known explicitly.

6. The **prime spectrum** of $\Gamma(G)$ can be explicitly described. $\text{Spec } \Gamma(G)$ connected.
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7. The correspondence $G \mapsto \Gamma(G)$ is a Green biset functor.
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7. The correspondence $G \mapsto \Gamma(G)$ is a Green biset functor.

8. The correspondence $G \mapsto \Gamma(G)^\times$ cannot be endowed with a structure of biset functor.
Conclusion

There are inclusions of Green biset functors $\mathcal{B} \hookrightarrow \mathcal{\Gamma} \hookrightarrow \mathcal{\Xi}$, and a projection $\pi: \mathcal{\Xi} \rightarrow \mathcal{\mathcal{B}}$, such that $\pi|\mathcal{\mathcal{B}} = \text{Id}_{\mathcal{\mathcal{B}}}$.

For any finite group $H$, and any field $k$, there are subfunctors $\mathcal{F}' \subset \mathcal{F} \subseteq k\mathcal{\Gamma}$ such that $\mathcal{\mathcal{F}}/\mathcal{\mathcal{F}}' \sim = \text{S}_H(k)$.

There are subfunctors $\mathcal{F}' \subset \mathcal{F} \subseteq k\mathcal{\Gamma}$ such that $\mathcal{\mathcal{F}}/\mathcal{\mathcal{F}}' \sim = \text{S}_K(V)$, for some $K$ and some $V \neq k$.

One can describe the lattice of ideals of $\mathcal{\mathcal{Q}}$.
There are inclusions of Green biset functors $B \hookrightarrow \Gamma \hookrightarrow \Xi$, and a projection $\pi: \Xi \rightarrow B$, such that $\pi|_B = \text{Id}_B$.

For any finite group $H$ and any field $k$, there are subfunctors $F' \subset F \subseteq k\Gamma$ such that $F/F' \sim S_{H,k}$.

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Serge Bouc (CNRS-LAMFA)
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Slice - Section
Pohang, March 28, 2011 17 / 17
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- There are inclusions of Green biset functors $B \hookrightarrow \Gamma \hookrightarrow \Xi$, and a projection $\pi : \Xi \to B$, such that $\pi|_B = Id_B$.
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- There are subfunctors $F' \subset F \subseteq k\Gamma$ such that $F/F' \cong S_{K,V}$, for some $K$ and some $V \not\cong k$.
- One can describe the lattice of ideals of $Q\Xi$. 