The Roquette category of finite p-groups and simple rational p-biset functors

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group G	

group G	$l_{\mathbb{Q}}(G)$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$

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$X_{3^3}^{+}$				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
$X_{3^3}^{+}$	6			

group G	$I_{\mathbb{Q}}(G)$	$I_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
$X^{+}_{3^3}$	6	11		

group G	$I_{\mathbb{Q}}(G)$	$I_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
$X_{3^3}^{+}$	6	11	$\mathbb{Z}/2\mathbb{Z}$	

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
$X_{3^3}^{+}$	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$

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X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}				

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X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6			

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^\times(G)$	$D_t(G)$
X_{33}^+	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^-	6	11		

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
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GAP(64,63)

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$(C_4 \times C_4) \rtimes C_4$				

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$(C_4 \times C_4) \rtimes C_4$	20			

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X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
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$(C_4 \times C_4) \rtimes C_4$	20	28		

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GAP(64,68)				

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$C_2 \times \mathrm{GAP}(64,63)$				

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$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40			

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$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56		

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$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	

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$C_2 \times$ GAP(64,68)				

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X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times$ GAP(64,63)				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68			

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112		

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^\times(G)$	$D_t(G)$
X_{33}^+	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^-	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times$GAP(64,68)				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68			

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112		

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^\times(G)$	$D_t(G)$
X_{33}^+	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^-	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
$X_{3^3}^{+}$	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$X_{3^3}^{-}$	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^\times(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16			

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16		

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^\times(G)$	$D_t(G)$
X_{33}^+	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^-	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0

group G	$I_{\mathbb{Q}}(G)$	$I_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
GAP(64,137)				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
$X_{3^3}^{+}$	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$X_{3^3}^{-}$	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4\times C_4)\rtimes C_2)\rtimes C_2$				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4 \times C_4) \rtimes C_2) \rtimes C_2$	16			

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4 \times C_4) \rtimes C_2) \rtimes C_2$	16	16		

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4\times C_4)\rtimes C_2)\rtimes C_2$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4 \times C_4) \rtimes C_2) \rtimes C_2$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4 \times C_4) \rtimes C_2) \rtimes C_2$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4\times C_4)\rtimes C_2)\rtimes C_2$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
GAP(64,177)				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2\times((C_4\times C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2\times((C_4\rtimes C_4)\rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4\times((C_4\times C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4\times((C_4\rtimes C_4)\rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4\oplus(\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4\times C_4)\rtimes C_2)\rtimes C_2$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$(C_2\times D_{16})\rtimes C_2$				

group G	$l_{\mathbb{Q}}(G)$	$l_{\mathbb{C}}(G)$	$B^{\times}(G)$	$D_t(G)$
X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4 \times C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4 \rtimes C_4) \rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{10}$
$C_2 \times ((C_4 \times C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_2 \times ((C_4 \rtimes C_4) \rtimes C_4)$	40	56	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{20}$
$C_4 \times ((C_4 \times C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$C_4 \times ((C_4 \rtimes C_4) \rtimes C_4)$	68	112	$(\mathbb{Z}/2\mathbb{Z})^{20}$	$(\mathbb{Z}/4\mathbb{Z})^4 \oplus (\mathbb{Z}/2\mathbb{Z})^{48}$
$(C_2)^4$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$((C_4 \times C_4) \rtimes C_2) \rtimes C_2$	16	16	$(\mathbb{Z}/2\mathbb{Z})^{16}$	0
$(C_2 \times D_{16}) \rtimes C_2$	16			

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X_{33}^{+}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
X_{33}^{-}	6	11	$\mathbb{Z}/2\mathbb{Z}$	$(\mathbb{Z}/2\mathbb{Z})^5$
$(C_4\times C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
$(C_4\rtimes C_4)\rtimes C_4$	20	28	$(\mathbb{Z}/2\mathbb{Z})^{10}$	$(\mathbb{Z}/4\mathbb{Z})^2\oplus(\mathbb{Z}/2\mathbb{Z})^{10}$
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Such a subgroup S is called a genetic subgroup of G.

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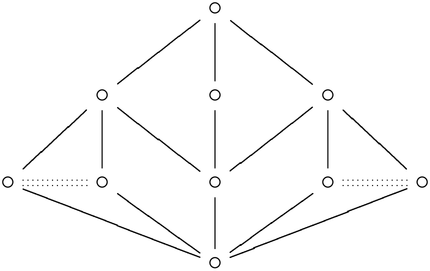

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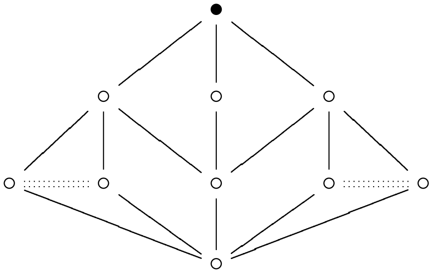

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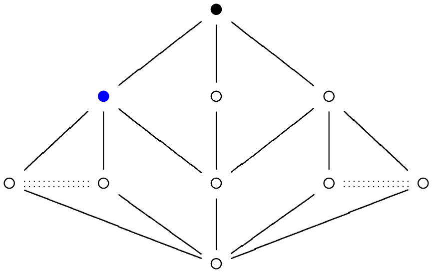

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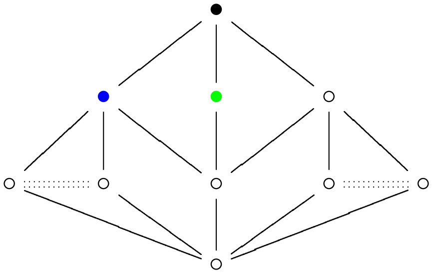

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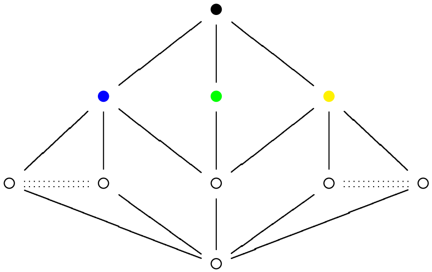

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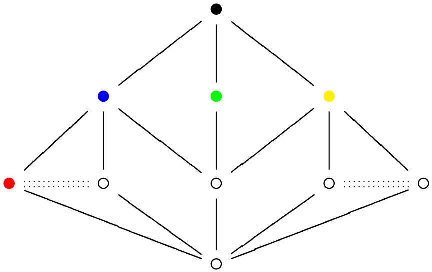

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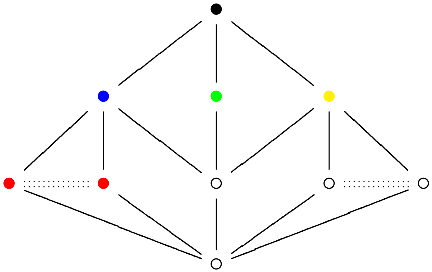

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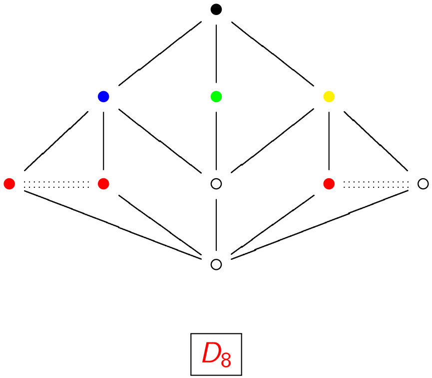

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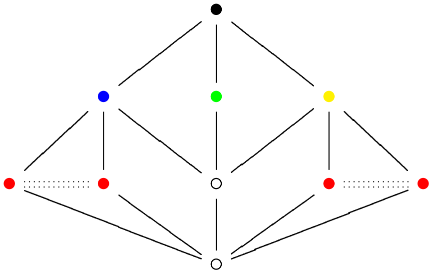

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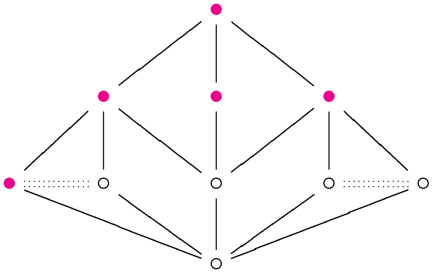

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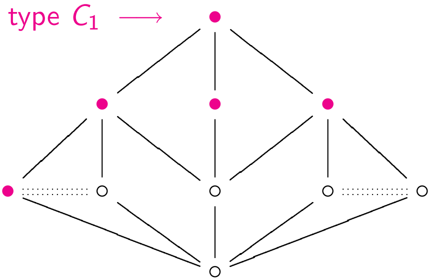

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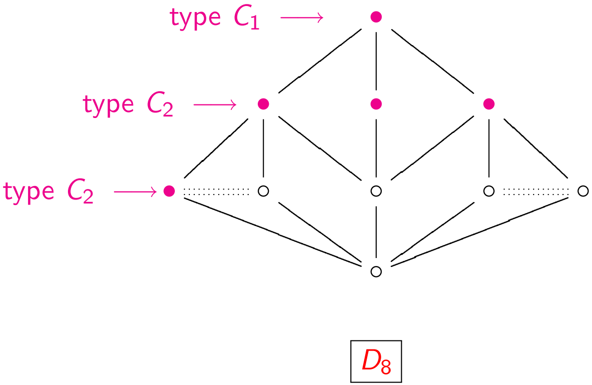

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The Roquette category

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- Morphisms are defined by bilinearity from

$$Hom_{\mathcal{R}_p}\big((P,e),(Q,f)\big)=f\,(B/B_\delta)(Q,P)\,e.$$

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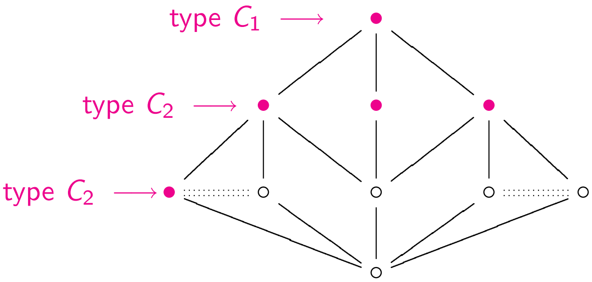

D_8

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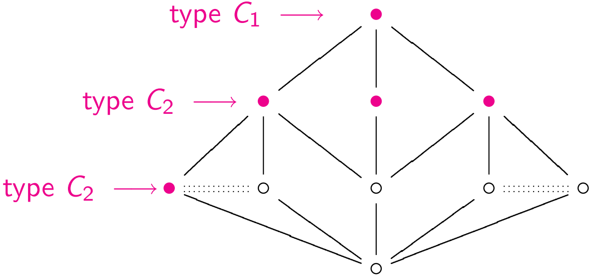

$\boxed{D_8}$

$D_8 \cong$

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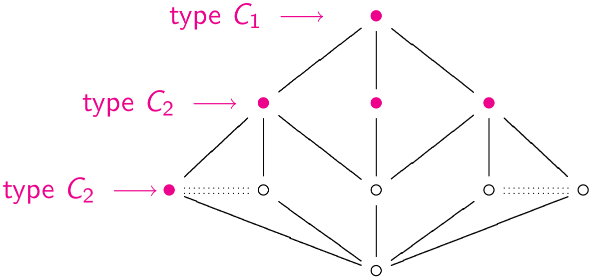

D_8

$$D_8 \cong \mathbf{1} \oplus 4 \cdot \partial C_2.$$

Tensor structure

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where $\nu_{P,Q} \in \mathbb{N} - \{0\}$ (explicit)

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Simple p-biset functors $\leftrightarrow S_{Q,V}$

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Simple p-biset functors	$\leftrightarrow$	$S_{Q,V}$
Which pair (Q, V)?	$\leftrightarrow$	$S_{Q,V}$ rational?

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Definition

Let k be a field, let C be a non-trivial cyclic p-group, and Z be its subgroup of order p.

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Definition

Let k be a field, let C be a non-trivial cyclic p-group, and Z be its subgroup of order p. A simple $kAut(C)$-module V is called primitive

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Definition

Let k be a field, let C be a non-trivial cyclic p-group, and Z be its subgroup of order p. A simple $kAut(C)$-module V is called primitive if $Tr_1^N(V) = \{0\}$, where $N = Ker\big(Aut(C) \to Aut(C/Z)\big)$.

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Theorem

Let p be a prime number, and k be a field. The simple rational p-biset functors are:

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- *the functors $S_{1,k}$.*

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Theorem

Let p be a prime number, and k be a field. The simple rational p-biset functors are:

- *the functors $S_{1,k}$.*
- *the functors $S_{C_{p^n},V}$*

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Theorem

Let p be a prime number, and k be a field. The simple rational p-biset functors are:

- *the functors $S_{1,k}$.*
- *the functors $S_{C_{p^n},V}$, where $n \geq 1$*

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- *the functors $S_{C_{p^n},V}$, where $n \geq 1$, V is a primitive simple $kAut(C_{p^n})$-module.*
- *when $p = 2$, the functors $S_{R,k}$*

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Theorem

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- *the functors $S_{1,k}$.*
- *the functors $S_{C_{p^n},V}$, where $n \geq 1$, V is a primitive simple $kAut(C_{p^n})$-module.*
- *when $p = 2$, the functors $S_{R,k}$, where R is a non-cyclic Roquette 2-group*

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Theorem

Let p be a prime number, and k be a field. The simple rational p-biset functors are:

- *the functors $S_{1,k}$.*
- *the functors $S_{C_{p^n},V}$, where $n \geq 1$, V is a primitive simple $kAut(C_{p^n})$-module.*
- *when $p = 2$, the functors $S_{R,k}$, where R is a non-cyclic Roquette 2-group, and k has characteristic 2.*