ERRATA FOR: REPRESENTATION THEORY; A HOMOLOGICAL ALGEBRA POINT OF VIEW

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page 7 line 16: $\varphi_i(m_i)$ instead of $\varphi(m_i)$ (thanks to Daniel Lopez Aguayo).

page 26 line 27: $\prod_{i=1}^{m} K_i$ (thanks to Daniel Lopez Aguayo).

page 36 line -10 ff: Let A be an algebra and let M be a finitely generated artinian A-module.... Further in the proof: Let $\mathcal{F} := \{T \leq M \mid M/T \text{ is semisimple}\}$. This set is not empty since M contains maximal submodules, which yield simple quotients. Indeed, let $\mathcal{S}$ be the set of proper submodules of M. This set contains 0, and is therefore not empty. If $(T_i)_{i\in I}$ is an increasing chain in $\mathcal{S}$, then the union is in $\mathcal{S}$, since else it contains the finite generating set, and therefore some element T_0 in the chain does, which contradicts the fact that T_0 is in $\mathcal{S}$. By consequence M has maximal submodules. Since M is artinian.... The hypothesis that M is finitely generated is necessary as shows the Prüfer group. A simpler proof can be given if M is supposed to be Noetherian.

page 49 lines 3 and 6: $\widehat{\beta^{-1}} : B \to A$ restricting to β^{-1} on S_B..... $\widehat{\beta} \circ \widehat{\beta^{-1}} = id_B$. (thanks to Daniel Lopez Aguayo).

page 50 line -2: Proposition 1.7.5 $\forall m \in M, n_1, n_2 \in N$ (thanks to Daniel Lopez Aguayo).

page 50 line -3: Proposition 1.7.5 $\varphi : N \otimes M \to B$ (thanks to Daniel Lopez Aguayo).

page 60 statement of Lemma 1.7.23: if M is a finite dimensional KH-module, then.... (thanks to Jorge Ledesma)

page 75 line 8, 9: extend α to... $\varphi(m_S) = n_S$ .... (thanks to Arthur Garnier).

page 92 statement of Lemma 1.8.27:

- The diagram is a pullback diagram if and only if it is commutative, γ induces an isomorphism on the kernels of α and of δ, and β induces a monomorphism on the cokernels of α and of γ.
- The diagram is a pushout diagram if and only if it is commutative, β induces an isomorphism on the cokernels of α and of δ, and γ induces an epimorphism on the kernels of α and of γ.

If the diagram is a pullback, let V be the kernel of the morphism induced by β on the cokernels of α, respectively δ. Then let W be the preimage of V in B. The embedding of W into B and the 0 map from W to C makes the diagram commutative, and hence there is a morphism from W to A composing with α to the identity on W. Hence W is a direct factor of A, and α is the identity on this direct factor. This contradicts the fact that W maps onto V.

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Conversely, consider the commutative diagram

$$\begin{array}{ccccccc} L & \overset{\lambda}{\hookrightarrow} & A & \overset{\alpha}{\longrightarrow} & B & \overset{\pi}{\twoheadrightarrow} & X \\ \simeq\downarrow\mu & & \downarrow\gamma & & \downarrow\beta & & \downarrow\overline{\beta} \\ K & \overset{\kappa}{\hookrightarrow} & C & \overset{\delta}{\longrightarrow} & D & \overset{\nu}{\twoheadrightarrow} & Y \end{array}$$

Let $X \xrightarrow{\rho} B$ and $X \xrightarrow{\sigma} C$ be morphisms with $\delta \circ \sigma = \beta \circ \rho$. Then $\overline{\beta} \circ \pi \circ \rho = \nu \circ \delta \circ \sigma = 0$ and since $\overline{\beta}$ is a monomorphism, $\pi \circ \rho = 0$. Hence ρ has image in $im(\alpha)$ and for all $x \in X$ there is $a \in A$ with $\alpha(a) = \rho(x)$. Since $\delta \circ \gamma(a) = \beta \circ \alpha(a) = \beta \circ \rho(x) = \delta \circ \sigma(x)$, we get $\gamma(a) - \sigma(x) = \kappa(k) = \kappa \circ \mu(\ell)$ for some $k \in K$ and $\ell \in L$, and hence Put $\phi(x) = a - \lambda(\ell)$. Then $\alpha \circ \phi(x) = \alpha(a) = \rho(x)$ and $\gamma \circ \phi(x) = \gamma(a) - \kappa(k) = \sigma(x)$. Hence the triangles in the diagram commute. Since μ is an isomorphism, ℓ is the unique possible modification on a. This shows the unicity. The fact that ϕ is a homomorphism is readily verified. Hence the diagram has the universal property of a pullback, and there is a pullback. Note that this simplifies and rectifies the proof on page 92 considerably. (thanks to Jin Zhang)

page 92 lines 16, 17, 18, 19: Moreover, $\lambda \circ \omega \circ \mu = \nu \circ \mu$, and hence $\gamma \circ \nu \circ \mu = \kappa \circ \mu = \gamma \circ \lambda$. Unicity implies $\gamma \circ \mu = \lambda$, and therefore $\lambda \circ \omega \circ \mu = \nu \circ \mu = \lambda \circ id$. Since λ is mono, $\omega \circ \mu = id$. (thanks to Arthur Garnier).

page 96 line 5, 10: We need that also for the first part of the statement that A is projective as B-modules. Then the exactness of line 5 of the proof is assured. Without this hypothesis the sequence $A \otimes_B X \to A \otimes_B Y \to A \otimes_B Z$ is not exact anymore. The second isomorphism should read as $Ext^i_A(M, \mathrm{Hom}_B(A, N)) \simeq \mathrm{Ext}^i_B(A \otimes_B M, N)$.

For the proof in the second case, let $\mathbf{P} \twoheadrightarrow M$ be a free resolution of M as A-module. Since A is a projective B-module, this is also a projective resolution of M as B-modules. Moreover, again since A is projective as B-modules, $A \otimes_B \mathbf{P}$ is a projective resolution of $A \otimes_B M$ as B-modules. Now, $\mathrm{Ext}^i_A(M, Hom_B(A, N)) \simeq H^i(\mathrm{Hom}_A(\mathbf{P}, \mathrm{Hom}_B(A, N)) \simeq H^i(\mathrm{Hom}_B(A \otimes_B \mathbf{P}, N) \simeq \mathrm{Ext}^i_B(A \otimes_B M, N)$. (thanks to Mamadou Sene)

page 101 Definition 1.8.37: and let $R = \mathbb{Z}$ be the ring of integers.

page 114 line 22 (in the statement of Theorem 1.8.47) ... finite group, let $N \trianglelefteq E$, and denote $G := E/N$. Suppose....

page 116, statement of Lemma 1.9.1: Let A be an artinian algebra and let P be an indecomposable projective A-module. (thanks to Jin Zhang)

page 159 line -16: ...every B-module is relatively A-projective, and ....

page 159 line -2: If P is A-projective, hence a direct factor of some A^n, then P is a direct factor of $B \otimes_A P$, and $B \otimes_A P$ is a direct factor of $B \otimes_A A^n = B^n$. Hence P is B-projective. By Proposition 2.1.6 every B-module is relatively A-projective.

page 180 line -4ff: By Lemma 1.7.44, kG is isomorphic to

$$\bigoplus_{N_G(D)gN_G(D) \in N_G(D)\backslash G/N_G(D)} k(N_G(D)gN_G(D))$$

as $kN_G(D) - N_G(D)$-bimodule. Now, denoting $N := N_G(D)$,

$$\begin{array}{rcl} NgN & \xrightarrow{\beta} & (N \times N)/\left(\Delta(N \cap {}^gN)\right)^{(1,g)} \\ n_1 g n_2 & \mapsto & (n_1, n_2^{-1}) \end{array}$$

is well-defined and bijective. Indeed,

$$n_1 g n_2 = n_1' g n_2' \Leftrightarrow \tilde{n} := (n_1')^{-1} n_1 = {}^g(n_2' n_2^{-1}) \in N_G(D) \cap {}^gN_G(D),$$

and so $n_1' \cdot \tilde{n} \cdot g \cdot (\tilde{n}^g)^{-1} \cdot n_2' = n_1 g n_2$. However,

$$\beta(n_1 g n_2) \cdot \Delta(\tilde{n})^{(1,g)} = (n_1, n_2^{-1}) \cdot \Delta(\tilde{n})^{(1,g)} = (n_1 \tilde{n}, n_2^{-1}(\tilde{n}^g)) = \beta(n_1 \cdot \tilde{n} \cdot g \cdot (\tilde{n}^g)^{-1} \cdot n_2).$$

Hence,

$$kN_G(D)gN_G(D) \simeq k(N_G(D) \times N_G(D)) \otimes_{k(\Delta(N_G(D) \cap {}^g N_G(D)))^{(1,g)}} k$$

as $k(N_G(D) \times N_G(D))$-modules. Therefore, the vertex of $k(N_G(D)gN_G(D))$ is in the group $(\Delta(N_G(D) \cap {}^g N_G(D)))^{(1,g)}$.

The following now comes from the proof of Lemma 13.7.c in Alperin: Local representation theory; Cambridge University Press 1986. We claim that $\Delta(D)$ is not conjugate to a subgroup of $(\Delta(N_G(D) \cap {}^g N_G(D)))^{(1,g)}$. Indeed, let $(h_1, h_2) \in N_G(D) \times N_G(D)$ such that $\Delta(D)^{(h_1,h_2)} \leq (\Delta(N_G(D) \cap {}^g N_G(D)))^{(1,g)}$, then $\Delta(D)^{(h_1,h_2g^{-1})} \leq (\Delta(N_G(D) \cap {}^g N_G(D))) \leq \Delta(G)$. Hence, for any $d \in D$ we have $h_1 g h_2^{-1} \in C_G(D) \leq N_G(D)$, which implies $g \in N_G(D)$, a contradiction.

Therefore, the vertex of $k(N_G(D)gN_G(D))$ for $g \notin N_G(D)$ is different from of $\Delta(D)$. However, for $g = 1$ we get $kN_G(D)$ as direct factor of the restriction of kG as $kN_G(D) - N_G(D)$-bimodule. We observe now.... (Thanks to Erik Darpö.)

page 215 Lemma 2.9.3: line 2 of the lemma: Then, the mapping... (Of course, $\mu_p : A \to A$ is not additive)

page 260 line before Example 3.1.2: ...possible when dealing with sets rather than with classes. (thanks to Jin Zhang)

page 264 line -7: $A - Mod$ (thanks to Arthur Garnier)

page 265 line 19: F a contravariant functor. (thanks to Arthur Garnier)

page 266 lines 1ff: Proposition 3.1.18 The index of the coproduct is not consistent, and actually incorrect in this generality, throughout the statement and the proof. Thanks to Arthur Garnier for pointing out these inconsistencies. Here is a correction:

Proposition 3.1.18: *Let A be an algebra and let $\mathcal{C} = A - Mod$. Further let $(I, \leq)$ be a codirected system. Let M be the functor mapping $i \in I$ to the object M_i of $\mathcal{C}$ and $i < j$ to $\iota_{(i,j)} \in Hom_{\mathcal{C}}(M_i, M_j)$. If we define $M_{i,j} := M_i$ for every $i < j$ and φ by*

$$\begin{array}{ccc} \coprod_{i,j \in I; i<j} M_i & \longrightarrow & \coprod_{i \in I} M_i \\ M_{i,j} \ni m_i & \mapsto & \iota_{i,j}(m_i) - m_i \in M_j \oplus M_i \end{array}$$

we get that this defines a homomorphism with cokernel the colimit $colim_{i \in I} M_i$.

Proof. We first get a homomorphism

$$M_i \xrightarrow{\binom{\iota_{(i,j)}}{-\mathrm{id}}} M_j \oplus M_i.$$

The universal property of the coproduct induces a homomorphism

$$M_i \longrightarrow \coprod_{i \in I} M_i$$

and again by the universal property of the coproduct a homomorphism φ as requested. The universal property of the cokernel implies the universal property of the colimit. ∎

page 267 line 11: the argument at the very left and the very right is $F(f)$ and not just f. (thanks to Arthur Garnier)

page 280 statement of Definition 3.3.8 line 5 of the definition: an inflation is a morphism which occurs as the second components of a conflation. (thanks to Jin Zhang)

page 283 statement of Lemma 3.3.13: ...Then the functor $Hom_A(M,-)$ commutes with arbitrary colimits (not only on inductive systems) if and only if M is finitely presented. (thanks to Pooyan Moradifar)

page 283 statement of Definition 3.3.14: *compact* if $Mor_{\mathcal{C}}:\mathcal{C}\to\mathbb{Z}-Mod$ commutes with arbitrary colimits. (thanks to Pooyan Moradifar)

page 284 line 4: the proof of Lemma 3.3.13 remains valid using $N=\mathrm{colim}_{i\in I}N_i$ instead.

page 284 line 11, 12: Suppose now that $Hom_A(M,-)$ commutes with arbitrary colimits. Now, M is the colimit...

page 297: Lemma 3.4.10: ... If two of the three left most vertical morphisms are isomorphism....

page 301 end of line -7: $H(\varphi)(m+d_M(M)):=$.... (thanks to Jin Zhang)
page 301 end of line -3: $\ker(\varphi_M)$. (thanks to Jin Zhang)

page 302 line -11: ... $d_N^{(i-1)}$.... (thanks to Arthur Garnier)

page 303 line -2 f: $Z_X\to X\leftarrow\widetilde{X}$ gives a morphism $Z_X\xrightarrow{\zeta}Z_Y$ in the homotopy category making the diagram ... (thanks to Gustina Elfiyanti)

page 310 Lemma 3.5.21: truncation $\tau_{\leq m}$ yields objects in C^+ and $\tau_{\geq m}$ yields objects in C^- (thanks to Arthur Garnier)

page 312 Proposition 3.5.25: Let $\mathcal{A}$ be an additive subcategory of an abelian category (thanks to Arthur Garnier)

page 313 line 3 after Definition 3.5.26: onto the first component

page 337 last line: $Hom_{K^-(\mathcal{A})}(pX,Y)$... (thanks to Jin Zhang)

page 349: Proposition 3.6.14: For group algebras the standard symmetrising form has the property that $\psi(1)=1$. We do not claim here that the basis $\mathcal{B}$ is formed by paths only. In particular, the form is not described by the values on the socle elements only.

page 357 first line at the end: ....$Hom_{K^-(\mathcal{A})}(pX,Y)$ (thanks to Jin Zhang)

page 389 line 12: the action of A on the right of $Hom_A(M,A)$ is given by $(f\cdot a)(m)=f(m)a$ for all $f\in Hom_A(M,A)$ and $a\in A$. Setting $(f\cdot a)(m)=f(am)$ will not produce an A-linear map $f\cdot a$ again, unless A is (for example) commutative. (Thanks to Benjamin Sambale.)

page 390 line 3: 1_A

page 403: Proposition 4.4.4: The statement of the proposition is false as presented, even if one assumes the additional hypothesis $gcd(|H|,|N|)=1$. (Thanks to Benjamin Sambale.)

page 417, statement of Lemma 4.5.6: Let K be a field and let A be a finite dimensional self-injective K-algebra. (thanks to Jin Zhang)

page 446: Definition 5.3.5: Let M be a finitely generated $B-A$-bimodule, and let N be a finitely generated $A-B$-bimodule such that....

This hypothesis is coherent with Lemma 5.3.1. We need M to be finitely generated, in order to get a functor $M\otimes_A-:A-\underline{mod}\to B-\underline{mod}$ between the stable categories of finitely generated modules. Indeed, if X is a quotient of A^n, and M is a quotient of $(B \otimes_K A^{op})^m$, then $M \otimes_A X$ is a quotient of $(B \otimes_K A^{op})^m \otimes_A A^m = B^{nm}$.

I am grateful to Serge Bouc for pointing out this mistake.

page 449: Proposition 5.3.11: line 4 of the proposition: .... $A \otimes_K A^{op}$-module, provided that P is projective as left A-module.
line 6 of the proposition: .... $A \otimes_K A^{op}$-module, provided that P is projective as right A-module. (thanks to Fernando Muro).

The additional hypothesis is used in Step 2 to ensure that $X \otimes_A Q^\bullet$ is still exact. For this, it is marked in the proof that P is right projective, hence $Q^\bullet$ is split as right modules. But in the following line we need $Q^\bullet$ to be split as complex of left modules. Proposition 5.3.11 is used in 5.3.13, and in 5.3.17. In both cases this additional hypothesis is verified. In 5.3.17 this follows from the fact that the algebras considered there are self-injective.

page 452: A similar condition as in Definition 5.3.5 could be imposed in Definition 5.3.15.

In both cases one may also consider the stronger condition that M is finitely generated projective as A-module and as B-module, and N is finitely generated projective as A-module and as B-module. Then M and N are compact and the corresponding functors have better properties (cf Lemma 3.3.13). Working with infinite dimensional algebras it seems to be useful to include conditions on the behaviour of functors under colimits.

page 508: In the diagram for $P(3,2)$, the entry $(3,1)$ should be replaced with $(2,1)$. (thanks to Klaus Lux)

page 510: Arrows in the quiver are going the opposite direction to the arrows in the ordering on page 506. (thanks to Klaus Lux)

page 511: The first relation of type 2 should have $\alpha_2^1\alpha_2^2$ replaced with $\alpha_2^2\alpha_1^2$. The first relation of type 3 should be $\alpha_2^1\alpha_2^2\alpha_2^1$. (thanks to Klaus Lux)

page 513: statement of Proposition 5.10.11: ... which admits only one isomorphism class of simple A-modules.... (thanks to Erik Darpö).

page 557 line -11: ... if $F : B - Mod \longrightarrow A - Mod$ is an equivalence... (thanks to Jin Zhang)

page 593 line -13: An argument is missing why an equivalence $D^b(A-mod) \simeq D^b(B-mod)$ maps the regular module ${}_BB$ to a tilting complex. We basically follow Rickard's argument. Since A and B are Noetherian, $D^b(A-mod) \simeq K^{-,b}(A-proj)$ and $D^b(B-mod) \simeq K^{-,b}(B-proj)$. Let hence $F : K^{-,b}(B-proj) \longrightarrow K^{-,b}(A-proj)$ be an equivalence of triangulated categories with quasi-inverse G. We may assume that

$$G(A) = Q^\bullet = \ldots \longrightarrow Q^0 \longrightarrow Q^{-1} \longrightarrow \ldots \longrightarrow Q^{s+1} \longrightarrow Q^s \longrightarrow 0 \longrightarrow \ldots$$

and $H^n(Q^\bullet) = 0$ for $n > 0$. Let

$$P^\bullet = F({}_BB) = \ldots \stackrel{d^{t+1}}{\longrightarrow} P^t \stackrel{d^t}{\longrightarrow} P^{t-1} \stackrel{d^{t-1}}{\longrightarrow} \ldots$$

and let $\check{P} := \tau_{\geq -s+1}P^\bullet$ be the stupid truncation of $P^\bullet$ in degree $-s+1$. In other words, $\check{P}$ coincides with $P^\bullet$ in degrees higher or equal to $-s+1$. Now, there is a natural map $\tau_{\leq t}\check{P} \stackrel{\iota_t}{\longrightarrow} \tau_{\leq t+1}\check{P}$ for all t. In general our category $K^{-,b}(A-proj)$ does not allow the construction of countable colimits, but this particular inductive system has a colimit $\check{P}$:

$$\check{P} = \mathrm{colim}_{t \geq -s+1}(\tau_{\leq t}\check{P})$$

by taking iterated cones. Indeed,

$$\tau_{\leq t+1}\check{P} = \text{cone}(P^{t+1}[t] \stackrel{(d^{t+1},0,0\dots)}{\longrightarrow} \tau_{\leq t}\check{P})$$

and hence

$$G(\tau_{\leq t+1}\check{P}) = G(\text{cone}(P^{t+1}[t] \to \tau_{\leq t}\check{P})) = \text{cone}(G(P^{t+1})[t] \to G(\tau_{\leq t}\check{P})).$$

Let us verify the universal property of a colimit. Take $X = (X, \partial_X)$ a complex in $K^{-,b}(B-proj)$ and let $\varphi_t : \tau_{\leq t}\check{P} \longrightarrow X$ be maps in $K^{-,b}(B-proj)$ so that $\varphi_{t+1} \circ \iota_t = \varphi_t$. Hence, there are maps $h_u : P^u \longrightarrow X^{u+1}$ for $1 \leq u \leq t$ such that

$$\varphi_{t+1} \circ \iota_t|_{P^u} = \varphi_t|_{P^u} + \partial_X^{t+1} \circ h_u + h_{u-1} \circ d^u$$

as morphisms of modules, giving an equality of morphisms of complexes $\tau_{\leq t}\check{P} \longrightarrow X$. But then, putting $h_{t+1} = 0$ we define

$$\varphi'_{t+1}|_{P_u} := \varphi_{t+1}|_{P_u} - \partial_X^{u+1} \circ h_u - h_{u-1} \circ d^u.$$

Then φ'_{t+1} is homotopy equivalent to φ_{t+1}, but we get furthermore $\varphi'_{t+1}|_{P_u} \circ \iota_t|_{P_u} = \varphi_t|_{P_u}$, and hence $\varphi'_{t+1} \circ \iota_t = \varphi_t$ as morphisms of complexes. We replace φ_{t+1} by the (homotopy equivalent) φ'_{t+1}. Taking appropriate representatives in the homotopy equivalence class of φ_t, by induction we can assume that the equations $\varphi_{t+1} \circ \iota_t = \varphi_t$ hold as morphisms of complexes. Let now $\varphi_t : \tau_{\leq t}\check{P} \longrightarrow X$ be a family of morphisms such that $\varphi_{t+1} \circ \iota_t = \varphi_t$. By the above we may assume that this holds in the category of complexes. Then, there is a morphism of complexes $\varphi : \check{P} \longrightarrow X$ such that $\varphi \circ \lambda_t = \varphi_t$, where $\lambda_t : \tau_{\leq t}\check{P} \longrightarrow \check{P}$ is the natural morphism. Indeed, $\varphi|_{P^u} = \varphi_t|_{P^u}$ for $t >> u$ defines such a morphism. Let $\psi : \check{P} \longrightarrow X$ be another morphism such that $\psi \circ \lambda_t = \varphi_t$. Then, $\psi - \varphi$ is homotopy equivalent to 0. Hence, we may assume that $\varphi_t = 0$ for all t and have to show that then ψ is homotopy equivalent to 0. Both X and $\check{P}$ have bounded homology, and so there is $k > 0$ such that $H^t(X) = 0 = H^t(\check{P})$ for all $t > k$. Since $\psi \circ \lambda_t = \varphi_t$ for all t, we choose $t = k+2$ and know that we may modify ψ by some homotopy (in degrees smaller than $k+2$, and 0 in higher degrees) such that the modified ψ satisfies $\psi(P^u) = 0$ for all $u < k+2$. We can hence assume that $\psi(P^u) = 0$ for all $u < k+2$. Consider $\tau_{\geq k}\check{P} =: M^\bullet$ and $\tau_{\geq k}X =: N^\bullet$. The objects $M^\bullet$ respectively $N^\bullet$ have homology M, respectively N, concentrated in degree k only. Moreover, ψ induces the 0 homomorphism $M \stackrel{0}{\longrightarrow} N$, and ψ is a lift $M^\bullet \stackrel{\psi}{\longrightarrow} N^\bullet$ of the 0 morphism $M \stackrel{0}{\longrightarrow} N$. Lemma 3.5.16 shows that ψ is homotopy equivalent to 0.

By induction on t, the homology of $G(\tau_{\leq t}\check{P})$ is concentrated in strictly positive degrees, and since P^{t+1} is projective, hence a direct factor of a finitely generated free module, GP^{t+1} is a direct factor of a finite direct sum of $Q^\bullet$. Since $Q^\bullet[t]$ is concentrated in degrees greater than $t+s > 0$ we are done. Moreover, $G(\check{P}) = \text{colim}_{t \geq -s+1} G(\tau_{\leq t}\check{P})$ again by taking iterated cones, since G is an equivalence and the universal property is hence preserved. Hence $G(\check{P})$ has homology concentrated in strictly positive degrees only. But

$$\begin{aligned} 0 = H^0(G(\check{P})) &= Hom_{K^{-,b}(B-proj)}({}_BB, G(\check{P})) \\ &\simeq Hom_{K^{-,b}(A-proj)}(F({}_BB), \check{P}) \\ &= Hom_{K^{-,b}(A-proj)}(P^\bullet, \check{P}) \end{aligned}$$

The natural map $P^\bullet \to \check{P}$ can be completed to a distinguished triangle

$$\hat{P} \longrightarrow P^\bullet \stackrel{0}{\longrightarrow} \check{P} \longrightarrow \hat{P}[1]$$

and hence $\hat{P} \simeq P^\bullet \oplus \check{P}[-1]$ for a bounded complex $\hat{P}$ of projectives. Therefore, $P^\bullet$ is isomorphic to a direct factor of a bounded complex of projectives. Since $Ext^i_B(B,B) = 0 \simeq Hom_{K^b(A-proj)}(P^\bullet, P^\bullet)$ for all non zero i, we get that $P^\bullet$ is a tilting complex. We are done. (Thanks to Henning Krause for reminding me that I have lost sight of this question.)

page 602 line -6: Corollary 6.6.4 is incorrect as stated. One needs the additional hypothesis $Ext^1_A(H_0(X), H_1(X)) = 0$. Without this hypothesis a counterexample is given by the tilting complex page 678 line -6. Indeed, let $T^{(2)} := X_2 \otimes_A A$ and let $n = 3$. Then $H_0(T^{(2)})$ is semisimple of dimension 2, corresponding to the endomorphisms of the simples $S_1 := P_1/rad(P_1)$ and $S_3 := P_3/rad(P_3)$. $H_1(T) = P_2 \oplus U_{3,2} \oplus U_{1,2}$, where $U_{i,j}$ is the 2-dimensional uniserial module with top $P_i/rad(P_i)$ and socle $P_j/rad(P_j)$. Now, $Ext^1_A(S_3, U_{1,2}) \neq 0$ since $rad(P_2)$ is indecomposable and has a submodule V isomorphic to $U_{1,2}$ with $rad(P_2)/V \simeq S_3$. The proof of Corollary 6.6.4 affirms that this extension group is 0, which is not correct. In the proof of Corollary 6.6.4 lines 4 to 15 should be erased.

page 614 line 1: ...and let A and B be two derived equivalent Noetherian.... (thanks to Jin Zhang)

page 678 line -6: The proof can be given similar to the proof in Section 6.7.1.

I thank Yuya Mizuno for pointing out that there are problems at this point and with Corollary 6.6.4.