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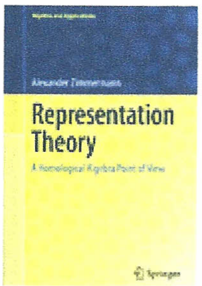
Representation Theory: A Homological Algebra Point of View



Alexander Zimmermann

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[Reviewed by Michael Berg, on 10/16/2014]

This is an algebra book — and how! The author's intent is to provide an obviously very serious "introduction to the representation theory of finite groups and finite dimensional algebras via homological algebra." The seriousness of his intent is already conveyed by his inclusion of Brauer theory (in the context of modular representations), Morita theory, and the certainly still pretty youthful theory of triangulated categories in the orbit of the book's almost 700 pages. Indeed the categorical point of view, which is indispensable of course, dominates the book, particularly its latter half or so, and although Zimmermann takes care to lay the foundations, it's pretty austere for the non-initiate.

As the book's subtitle provides, the general tenor of it all is homological algebra and the aforementioned foundational material occupies the first chapter of the book: it's a combination of the fare in a solid graduate algebra course (Jacobson-style, say) and, for example (*par excellence*: this is one of my favorite books) Cartan-Eilenberg; nowadays, Weibel would probably get the nod, but my heart belongs to Bourbaki and the former text. It should be noted that Zimmermann closes his introductory chapter (of about 150 pages) with stuff on "algebras defined by quivers and relations" — certainly this should suggest to the reader what he's in for.

But there's much more, of course. After this much-needed solid introduction, laying of the foundation for what's to come, it's a quick ascent: modular representation theory, triangulated categories, Morita theory, as already indicated, and then, going even higher: stable module categories, and derived equivalences (where the influence of algebraic topology is strongly felt: witness, e.g. section 6.3 on strong homotopy action). These last two chapters include such things as syzygies, Nakayama algebras, Hochschild (co)homology, a number of things Grothendieck, and a final section on Picard groups of derived module categories: again a clear indication of what league we're playing in if we join this game — the most modern algebra, replete with very significant and deep results of relatively recent vintage, plus interplay with algebraic topology and algebraic geometry (if somewhat sub rosa).

The following statement from Zimmermann's Preface comprises something of a characterization of the book's *raison d'être*: "The bridge from the representation theory of algebras to the representation theory of groups via homological algebra was fully established in 1989, when Rickard proved a Morita theory for derived categories and Broué pronounced his most famous abelian defect conjecture" (regarding the latter, see p. 665ff.). To convey the flavor of all this, and to underscore the seriousness of the algebra Zimmermann is dealing with, here is the aforementioned conjecture of Broué (loc.cit., p. 666):

> "*Let k be an algebraically closed field of characteristic $p > 0$, let G be a finite group, let P be an abelian Sylow p-subgroup of G and let $H = N_G(P)$. Let B_G be the principal block of [the group algebra] kG and let B_H be the principal block of kH. Then there should be an equivalence of triangulated categories $D^b(B_G) \cong D^b(B_H)$.*"

Thus, it is clearly the case that Zimmermann's book is geared to initiates and serious algebraists aiming at research in the indicated area. It is clearly a labor of love and fine scholarship, and should succeed in providing guidance and instruction in a most interesting and intricate subject.

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