Thermodynamic formalism in non-hyperbolic dynamics

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We aim for an understanding of the thermodynamic formalism in the context of smooth non-hyperbolic dynamics. We start by investigating its consequences for some particular examples and follow a classical approach to analyze its basic pieces. In particular, we study homoclinic classes together with their thermodynamic properties. Focusing on a simple, by representative, example, we construct a local diffeomorphism that is a step skew product modeled over a horsehos map that are naturally asociated to a heterodimensional cylce. This cycle gives rise to a homoclinic class on which the diffeomorphism is topologically transitive and partially hyperbolic. It can be conveniently studied in terms of an iterated function system of interval maps that are genuinely non-contracting. Our examples have topologically a rich fibre structure. Moreover, they exhibit a rich phase transition in the pressure function (coexistence of equilibrium states with positive entropies) that is associated to the Lyapunov exponents in the central direction. This phase transition is a consequence of a gap in the spectrum of central exponents. We further discuss shadowing-like properties and derive minimality of the IFS. This is a joint work with L. Diaz (PUC Rio de Janeiro) and M. Rams (IM PAN Warsaw).